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Finite elements for Gradient and Cosserat elasticity: a penalty approach

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ABSTRACT

Modelling the effects of material microstructure has attracted significant interest in recent years. A landmark contribution is due to Mindlin [1], who considered degrees of freedom of micro-deformation ψ_{ij} at each material point, in addition to the three displacements. In Mindlin's elasticity with microstructure, energy is stored due to strain ϵ_{ij}, relative deformation $\gamma_{ij} = u_{j,i} - \psi_{ij}$ (where $u_{j,i}$ the displacement gradient) and the micro-deformation gradient $\kappa_{ijk} = \psi_{jk,i}$, leading to an expression for the strain energy density with 18 material constants in the isotropic case [1]. The resulting 12 governing equations can be discretised using standard finite element techniques and shape functions [2].

Two other well-known approaches are the gradient approach, where higher order strain gradients are included in the strain energy density, and the Cosserat approach, where additional degrees of freedom of point rotation are considered along with the usual translational ones. Both theories result to additional, higher order terms in the governing equations and are, as a result, more complicated to treat numerically with the finite element method than classical elasticity. Gradient elasticity, in particular, contains strain gradient terms in the virtual work expression, leading to the requirement for C^1 interpolation if the displacement field only is discretised. Appropriate elements exist and perform very well [3][4], however the approach is often seen as complicated and/or computationally costly. Cosserat elasticity is simpler to implement, as the additional terms can be treated using standard shape functions.

It has been shown that both gradient elastic and Cosserat continua are limiting cases within Mindlin's framework [1]. In particular, the gradient elastic medium corresponds to the limit where micro-deformations coincide with the displacement gradient, i.e. where relative deformation vanishes. Similarly, Cosserat elasticity corresponds to the limit where the symmetric part of the micro-deformation (i.e. the micro-strain) vanishes, but not the antisymmetric part, which gives rise to the Cosserat rotation.

Here we exploit these facts by producing appropriate combinations of material parameters that enforce the above limits in an approximate manner, and then using the finite element discretisation of Mindlin's elasticity with microstructure to produce numerical solutions for the corresponding gradient elastic [2] and Cosserat solids. This approach is equivalent to using a penalty method, as some of the material

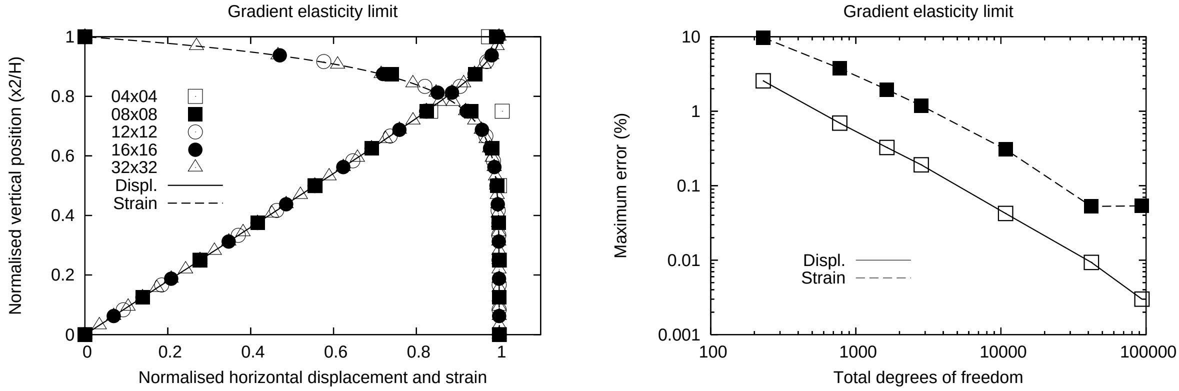

Figure 1: Shearing a gradient elastic layer: maximum solution error vs mesh refinement.

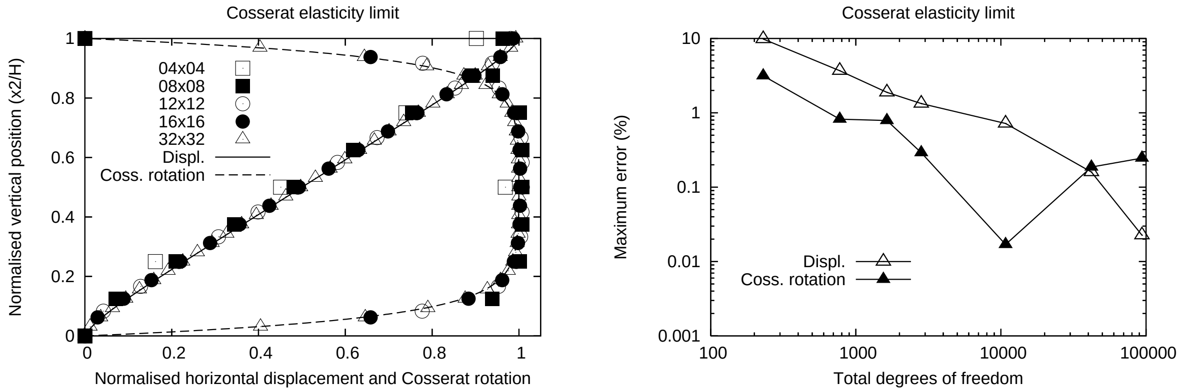

Figure 2: Shearing a Cosserat elastic layer: maximum solution error vs mesh refinement.

parameters take the role of expressing the relevant constraints. Good quality approximate solutions are obtained, and errors of less than 1% can be achieved in benchmark boundary value problems (e.g. Figures 1 and 2)

The presented approach gives an arguably simpler to implement alternative to the more complicated and computationally expensive treatments commonly used for gradient elasticity. In addition, it provides a unified framework within which good quality numerical solutions can be produced for the three different types of continuum (elastic with microstructure, gradient and Cosserat) by a single finite element code.

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