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Parameters identification for the nonlinear time-dependent model of rubber bushings in multibody simulations

Beatrice Bourgeteau[1, 2], Denis Aubry[1] and Guillaume Puel[1]

[1] MSSMat (Ecole Centrale Paris / CNRS UMR 8579)
Grande Voie des Vignes
92290 Châtenay Malabry
France
{beatrice.bourgeteau, denis.aubry, guillaume.puel}@ecp.fr

[2] PSA Peugeot Citroën
18, rue des Fauvelles
92 825 La Garenne-Colombes
France

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ABSTRACT

Rubber bushings are linking parts which are extensively used in vehicle chassis to reduce noises and vibrations. Although their damping properties improve the vibratory comfort they can still weaken the driver comfort e.g. their properties may modify the chassis behaviour and influence the vehicle steering behaviour itself. Moreover differences between bushings properties, which have not been taken into account until now in multibody simulation, have been highlighted through experiments. The model used here improves the classical description of rubber bushings behaviour within the small deformation framework and its resulting influence on the vehicle performances. It allows the modelling of a non linear time-dependent behaviour, especially transient behaviour with very few parameters. This model is also able to describe the viscoelastic, hysteretic, as well as the relaxation behaviour of the bushing.

Taking into account the above considerations, the accurate determination of the parameters given a set of experimental measurements is obviously a critical issue for a reliable and satisfactory use of the model. The objective of this presentation is thus to introduce a robust identification method which can incorporate simultaneously transient and frequency measurements and which is briefly described below.

The nonlinear viscoelasticity of the model implies that the differential system to solve is highly nonlinear and even not explicit with respect to the time derivative and has the following expression:

$$\begin{cases} F(y'(t),\, y(t),\, t,\, p) \;=\; 0 \\ y(0) \;=\; y_0(p) \end{cases} \tag{1}$$

where y stands for the state of the model, *(.)'* the time derivative and p the set of parameters to be determined. Given the parameters p, the state y is a function $y(t,p)$ of time and parameters.

It is assumed that experimental measurements and velocities have delivered values of subsets of the state $Ay_d(t\ )$ and $By_d'(t\ )$, where A and B are suitable projection operators. The parameters p have to be chosen so that the following cost function is minimized:

$$J(p) = \frac{1}{2}\int_0^T (\left|A(y - y_d)\right|^2 + \left|B(y' - y_d')\right|^2)dt + \frac{1}{2}(C(p - p_0), p - p_0) \tag{2}$$

where p_0 consists of typical values of the sought parameters, in order to regularize the inverse problem, with C a weighted regularisation matrix to be chosen carefully.

In order to estimate the derivative δJ of the cost function with respect to the parameters, a generalized adjoint state z is introduced which is defined here in this special case by:

$$\begin{cases} \left(D_{y'}F^T z\right)' - D_y F^T z = -A^T A(y - y_d) + B^T B(y'' - y_d'') \\ z(T) = 0; y'(0) = 0 \end{cases} \tag{3}$$

where D_y and $D_{y'}$ are the derivatives with respect to y and y' respectively and $(.)^T$ is the transposition. Then the derivative of the cost function may be computed readily and the optimality equation obtained:

$$C(p - p_0) + \int_0^T D_p F^T z\ dt = 0 \tag{4}$$

where D_p is the derivative with respect to the parameters p. This last equation is interesting in that it shows that the optimal parameters are indeed a weighted time average of the derivative of the state equation with respect to the parameters multiplied by the adjoint state.

The nonlinear differential system (1)-(3)-(4) which involves the three unknowns $\{y,z,p\}$ can be solved by an iterative process. The talk will present several theoretical and practical aspects and illustrations of the proposed approach either at the model or multibody level.

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