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HOMOGENIZED MODEL OF BONE POROELASTICITY AND DEFORMATION INDUCED MICROFLOW

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ABSTRACT

The bone mechanotransduction is an important phenomenon involved in many biological processes in the living bone. Due to porous structure of the cortical bone, any dynamic mechanical loading causes fluid flow through the hierarchy of pores arranged in the bone units called osteons. These are distributed almost periodically and involve pores of different sizes; from the Haversian canal forming the axis of each osteon the fluid can enter the system of lacuno-canalicular porosities which drains the porous ultrastructure of the collagen apatite matrix which, thus, presents the second level of porosity.

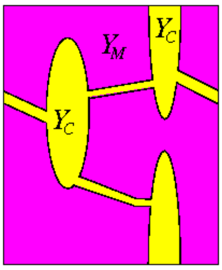

Fig. 1: *The decomposition of Y (a 2D scheme).*

We consider the bone tissue as the double porous medium, see e.g. [2], fluid saturated medium with periodic structure. The hierarchy of pores is considered at the level of osteons, canalicular network and solid porous matrix. It is represented by the representative periodic cell $Y = \Pi_{i=1}^3]0, b_i[$ with the decomposition $Y = Y_M \cup Y_C \cup \partial_C Y_M$, where $\partial_C Y_M$ is the interface, see Fig. 1; domain Y_C reflects the system of the connected Haversian and Volkmann canals relevant to the meso-scale, generating a periodic lattice, whereas Y_M is occupied by the porous matrix involving the canalicular network which is represented using the anisotropic elasticity E_{ijkl} and permeability κ_{ij}^M, tensors.

Following the ideas reported in [1], the homogenization method is applied to the Biot model of the porous deforming medium, where the elasticity, permeability and the Biot parameters defined locally describe the bone ultrastructure. The dual porosity is accounted for by scale dependent permeability in Y_M, thus

$$K_{ij}(y) = \chi_C(y)\kappa_{ij}^C(y) + \chi_M(y)\varepsilon^2\kappa_{ij}^M(y) \ , \quad y \in Y,$$

where $\varepsilon > 0$ is the scale parameter, χ_D is the characteristic function of domain Y_D, $D = C, M$.

Using the unfolding method of homogenization [1], we obtained the macroscopic model describing the deformation induced flow [1,4] in the porous viscoelastic skeleton. The displacements $\boldsymbol{u}$ and the time-integrated pore pressure P satisfy the "macroscopic" problem, involving the homogenized visco-elasticity, $\mathcal{E}_{ijkl}$, $\mathcal{H}_{ijkl}$, the homogenized Biot coefficients, $\mathcal{B}_{ij}, \mathcal{F}_{ij}$, the homogenized Biot moduli, $\mathcal{M}, \mathcal{G}$, and the homogenized permeability of the interconnected porosities, $\mathcal{C}_{ij}$: for a.a. $t \in]0,T[$ find $\boldsymbol{u} \in V$ and $P \in Q$ (with $P(0) = 0$) such that

$$
\begin{aligned}
&\int_\Omega \mathcal{E}_{ijkl} e_{kl}(\boldsymbol{u}) e_{ij}(\boldsymbol{v}) + \int_\Omega \int_0^t \mathcal{H}_{ijkl}(t-\tau) e_{kl}(\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{u}(\tau))\, d\tau\, e_{ij}(\boldsymbol{v}) \\
&- \int_\Omega (\mathcal{B}_{ij} + \mathcal{F}_{ij}(0_+)) \frac{\mathrm{d}}{\mathrm{d}t} P\, e_{ij}(\boldsymbol{v}) - \int_\Omega \int_0^t \mathcal{F}_{ij}(t-\tau) \frac{\mathrm{d}}{\mathrm{d}\tau} P(\tau)\, d\tau\, e_{ij}(\boldsymbol{v}) = \int_\Omega \boldsymbol{f} \cdot \boldsymbol{v} \;, \\
&\int_\Omega \mathcal{B}_{ij} e_{ij}(\boldsymbol{u})\, q + \int_\Omega \int_0^t \mathcal{F}_{ij}(t-\tau) e_{ij}(\frac{\mathrm{d}}{\mathrm{d}\tau}\boldsymbol{u}(\tau))\, d\tau\, q + \int_\Omega \mathcal{C}_{ij} \partial_j P \partial_j q \\
&\qquad + \int_\Omega \mathcal{M} \frac{\mathrm{d}}{\mathrm{d}t} P\, q + \int_\Omega \int_0^t \mathcal{G}(t-\tau) \frac{\mathrm{d}}{\mathrm{d}\tau} P(\tau)\, d\tau\, q = 0 \;,
\end{aligned}
$$

for all $\boldsymbol{v} \in V_0$ and $q \in Q_0$, where V, Q are admissibility sets for $\boldsymbol{u}$ and P, resp., while V_0, Q_0 are the associated test spaces. While $\mathcal{C}_{ij}$ is determined only by the permeability and the geometry of Y_C, the viscoelastic properties as well as the homogenized parameters of the Biot type are computed by the characteristic responses: the correctors of the macroscopic strains and of the pressure field. These correctors solve the *microscopic equations* governing the Darcy flow in the deforming porous matrix. They are defined in the representative periodic cell, see Fig. 1, and retain some features of the original Biot model which is the subject of the upscaling procedure. The fluid seepage is restricted just to the dual porosity in subdomain Y_M.

In the paper we discus the multiscale modeling of the compact bone poromechanics, the deformation induced flow in the hierarchy of the pores, regarding its specific structural geometry at several scales of heterogeneities. In the *numerical examples computed using our FE code* we illustrate some options of the modeling: **1)** quantification of the homogenized constitutive parameters (the viscoelasticity and permeability) for a given micro- and meso-structure; this allows for identification of the microstructure-related constitutive parameters (the elasticity and the permeability) using measurements at the meso-scopic scale; **2)** localization of strain, stress, pressure and diffusion velocity for a defined geometry (micro/meso scales) and for a given local macroscopic response.

The further research will pursue the fluid transport in bone pores and the associated stress generated streaming potentials [3], once the model is enriched by the convection–diffusion of dissolved charged species (cations, anions). Thus, the present model provides a framework for a systematic development of a microstructure-oriented model of bone tissue remodeling and growth.

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