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QUALITATIVE BEHAVIOR OF SOLUTIONS IN THE VICINITY OF TOOL SURFACE IN PLASTICITY THEORY FOR POROUS AND POWDER MATERIALS

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ABSTRACT

Various theories of rigid plastic solids provide an adequate description of material response in the modelling of manufacturing processes in powder metallurgy. In many cases neglecting the elastic portion of strain makes no essential effect on the final result. Efficient numerical methods can be developed for solving the corresponding boundary value problems to significantly reduce the computing time, as compared with traditional finite element methods for elastic-plastic constitutive equations. A typical example of such an approach is the numerical procedures based on the method of characteristics in the theory of rigid perfectly/plastic materials [1]. However, the system of equations for plastic flow of powder and porous materials is not always of the hyperbolic type [2]. Therefore, it is important to understand the qualitative behaviour of solutions in the vicinity of surfaces with special properties, which can cause difficulty with numerical integration of equations. One type of such surfaces is the interface between deforming material and rigid tool. Since the tool is supposed to be rigid, the in-surface strain rates vanish at sticking. This is a strong restriction on the velocity field and in many cases the regime of sticking is not compatible with other boundary conditions in rigid plasticity [3, 4]. In turn, the regime of sliding can lead to singular solutions such that the equivalent strain rate approaches infinity at the friction surface [4, 5]. However, this property of solutions depends on the constitutive equations chosen. Another possible difficulty with numerical integration of equations of rigid plasticity is the appearance of rigid zones where the system of equations is not complete. The purpose of the present study is to show qualitative features of rigid plastic solutions for porous and powder materials by means of a simple example.

The initial/boundary value problem considered consists of a planar deformation comprising the simultaneous shearing and contraction of a hollow cylindrical specimen of porous material. Symmetry in the circumferential direction dictates that all quantities are a function of the radial direction only. The outer cylindrical boundary contracts and is fixed against rotation, the radius of the inner boundary is constant and the boundary rotates, thereby inducing a shearing motion in the material. Two different regimes may

be identified, when the material sticks to the inner boundary and when the material slips at the boundary. The main questions addressed in the paper are: (*i*) Is it always possible to find the solution at sticking? and (*ii*) Is the solution singular at the friction surface?

The plasticity theory for porous materials proposed in [6, 7] is adopted. The theory is based on a pressure dependent yield criterion and its associated flow rule. It has been shown that the final equation for the radial velocity is written in the form

$$\frac{du_r}{dr} = \Phi(\theta, r) u_r \tag{1}$$

where θ is the porosity and Φ is the known function of its arguments. The porosity is also a known function of *r*. Thus (1) is a linear differential equation of first order. Since one of the boundary conditions requires that $u_r = 0$ at the inner radius, it is obvious that the solution to (1) is $u_r = 0$, unless the function Φ is of a special form. The latter case is studied in detail. It is concluded that in general a rigid zone exists at the beginning of the process. At this stage of the process $u_r = 0$ at a moving radius where the function Φ satisfies the conditions that ensure the existence of a non-trivial solution to equation (1) in the domain between the moving radius and outer radius of the cylinder (plastic zone). The domain between the moving radius and the inner radius of the cylinder remains rigid. The moving radius propagates to the inner radius of the cylinder. Once the plastic zone has occupied the entire domain between the inner and outer radii, the regime of sliding becomes possible. This case is also investigated in detail. In particular, an asymptotic representation of the velocity field in the vicinity of the inner boundary at sliding is found. It is also shown that the velocity field may be singular in the vicinity of the inner radius at sliding. It is believed that these qualitative features of the solution are common for a wide class of problems in the theory of rigid plasticity for porous and powder materials.

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