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A finite element formulation for wrinkling at finite strains based on energy minimization

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ABSTRACT

This paper is concerned with an efficient novel algorithmic formulation for wrinkling at finite strains. The distinguishing feature of this method is that every aspect is governed by variational principles. More precisely, inspired by a series of papers by Pipkin [1, 2], the parameters describing wrinkles or slacks, together with the unknown deformation mapping, are computed jointly by minimizing the potential energy of the considered mechanical system, cf. [3]. Furthermore, the wrinkling criteria are naturally included within the presented variational framework.

The proposed method allows to employ three-dimensional constitutive models without any additional modification, i.e., a projection in plane stress space is not required. Analogously to the wrinkling parameters, the non-vanishing out-of-plane component of the strain tensor results conveniently from relaxing the respective Helmholtz energy of the membrane. Additionally, the proposed framework is very general and does not rely on any assumption regarding the symmetry of the material, i.e., arbitrary anisotropic hyperelastic models can be consistently taken into account. In case of fully isotropic constitutive models, the efficiency of the presented algorithmic formulation can be significantly increased, since the wrinkling directions are known in advance.

The advantages associated with such a variational method are manifold. For instance, it opens up the possibility of applying standard optimization algorithms to the numerical implementation. This is especially important for highly non-linear or singular problems such as wrinkling. On the other hand, different minimization principles can often be coupled in a straightforward manner. As a prototype, the incrementally defined minimization principle associated with associative finite strain plasticity theory based on a multiplicative decomposition of the deformation gradient is considered. By combining the kinematics describing the elasto-plastic split with the deformation modes induced by wrinkling, a novel variationally consistently coupled model suitable for the numerical analysis of wrinkles in inelastic membranes is derived, see [4]. Analogously, to its purely elastic counterpart, all unknown variables such as the plastic part of the deformation gradient, the wrinkling parameters and the deformation mapping follow in a natural manner by minimizing an incrementally defined energy.

References

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