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# ASYMPTOTIC REPRESENTATION OF THE EQUIVALENT STRAIN RATE IN THE VICINITY OF MAXIMUM FRICTION SURFACES IN VISCOPLASTICITY

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## ABSTRACT

Conventional rigid viscoplastic material models are obtained as a generalization of the model of rigid perfectly/plastic solids based on the Mises yield criterion and its associated flow rule. An important property of the constitutive equations of the latter model is that the equivalent strain rate is represented by [1]

$$\xi_{eq} = D\big/\sqrt{s} + o\left(1\big/\sqrt{s}\right), \qquad s \to 0 \tag{1}$$

near maximum friction surfaces. Here $s$ is the distance from the surface and $D$ is the strain rate intensity factor. The maximum friction surface is defined by the condition that the friction stress at sliding is equal to the shear yield stress. The solution behaviour (1) can cause numerical difficulty because the velocity field is represented by non-differentiable functions. Some difficulties with numerical solutions in the vicinity of friction surfaces were reported in [2 – 4] where more general constitutive equations than those of rigid perfectly/plastic solids were used. Another important point is that the representation (1) is in qualitative agreement with experiment [4 – 6], assuming that $\xi_{eq}$ has a significant effect on material properties whose distribution in the vicinity of frictional interfaces was determined [4 – 6]. A method for predicting such distributions based on the value of $D$ was proposed in [7]. It is therefore of interest to extend the theory of singular solutions [1] to other material models. In particular, viscous effects should be important at high strain rates. However, the constitutive equations of conventional viscoplastic models require the regime of sticking at maximum friction surfaces and $\xi_{eq}$ in this case is not singular [8]. In particular, it results in non-convergence of viscoplastic solutions to the corresponding rigid perfectly/plastic solutions at the set of parameters that transform the constitutive equations of viscoplastic models into the constitutive equations of rigid perfectly/plastic model. Yet, such behaviour of viscoplastic solutions is a possible cause of numerical difficulty mentioned in [2]. Therefore, a class of viscoplastic models with a saturation stress was proposed in [9]. It was shown that the velocity field may be singular near maximum

friction surfaces. However, the type of singularity is in general different from (1). A disadvantage of that study is that the normal strain rates in a local Cartesian coordinate system associated with the friction surface are zero at the surface. It is known [1] that this condition may change the general qualitative asymptotic behaviour of $\xi_{eq}$. Therefore, in the present paper a boundary value problem free of this disadvantage is studied. The constitutive equations are the Mises yield criterion with the tensile yield stress, $\sigma_Y$, dependent of $\xi_{eq}$ and its associated flow rule. It is supposed that $\sigma_Y \to \sigma_s < \infty$ as $\xi_{eq} \to \infty$. It is shown that the solution behaviour in the vicinity of maximum friction surfaces is completely determined by the asymptotic behaviour of $\sigma_Y$ as $\xi_{eq} \to \infty$. The study is restricted to relations in the form

$$\sigma_Y = \sigma_s\left(1 - \phi_\infty \xi_{eq}^{-\beta}\right) + O\left(\xi_{eq}^{-\beta-\varepsilon}\right), \qquad \xi_{eq} \to \infty \tag{2}$$

with $\beta > 0$, $\phi_\infty > 0$ and $\varepsilon > 0$. It is shown that for $\beta > 2$ in (2) sliding is possible at maximum friction surfaces and, in this case, the asymptotic behaviour of $\xi_{eq}$ coincides with (1). In particular, the concept of strain rate intensity factor can be extended to viscoplastic models of this type. Difficulties that may occur in numerical calculation of such velocity fields can be overcome by introducing appropriate shape functions in finite elements near the friction surface.

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