8th. World Congress on Computational Mechanics (WCCM8)
5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

APPLICATION OF THE LOWER BOUND SHAKEDOWN THEOREM FOR THE ASSESSMENT AND DESIGN OF COMPOSITES

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Key Words: *Direct Methods, Shakedown- and Limit Analysis, Composites, Non-linear Optimisation.*

ABSTRACT

To determine load limits for mechanical structures under monotonous or variable loading beyond the elastic range of material behaviour, Direct Methods have been developed and successfully applied since many years. An essential element for their functioning is that dissipative effects occurring during inelastic deformation cause sufficiently well determined changes of internal parameters, which stabilize the process of deformation within certain limits. This applies not only to classical elastic-plastic structures characterized by separate convex plastic potentials but also to other dissipative processes such as e.g. deformation processes implying frictional and damaged material with limited unstable behaviour.

Subject of this paper is the relatively new area of application for Direct Methods to composite materials, where at least one phase has ductile properties in the up-mentioned sense. Composites are regarded here on a meso-level as inhomogeneous structures characterized by a pattern of periodicity as to the arrangement of the different phases such as matrix material and reinforcing fibres or particles. Then, using the concepts of representative volume elements and homogenisation, the load-limits for such materials under monotonous and, to a certain extent, under variable load conditions can be determined with the help of Direct Methods. Different physical effects such as plasticity, material damage and decohesional effects are included in the theoretical part of the paper.

The authors restrict their attention to lower-bound-methods. The presented numerical approach is based on a finite-element discretisation of the structured material, the knowledge of a purely elastic reference solution that ignores the dissipative effects occurring during the deformation process and a convex, non-linear numerical

optimisation procedure with non-linear constraints, which represent the generalised yield-condition. Objective function is the so-called load-factor α, which is the multiplier by which a normalised load domain can be magnified so that failure just does not occur.

The specific characteristics of the optimisation problem, which are non-linearity, convexity as well as large numbers of variables and subsidiary conditions, suggested the development of tailored optimisation procedures, which allowed drastically reducing the necessary CPU time for calculation of complex problems. These recent achievements will be presented on illustrative examples allowing a comparison with formerly applied procedures.

The last issue addressed is the combination of maximising the loading factor with to the optimal spatial arrangement of different phases of the composite, such as the density of reinforcing fibres in a mechanical component, under shakedown conditions. It will be shown that this problem involves a double optimisation, only one of which being of convex nature.

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