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An optimal fictitious domain method inspired by the extended finite element method

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ABSTRACT

We present a new fictitious domain approach inspired by the extended finite element method (Xfem) of Moës, Dolbow and Belytschko [4]. An optimal method is obtained thanks to an additional stabilization technique which is an adaptation of the one of Barbosa-Hughes [1]. Some *a priori* estimates are established and numerical experiments illustrate different aspects of the method. The presentation is made on a simple Poisson problem with mixed Neumann and Dirichlet boundary conditions. The extension to other problems or boundary conditions is quite straightforward. The stabilization proposed allows the method to converge whatever is the intersection of the domain with the mesh.

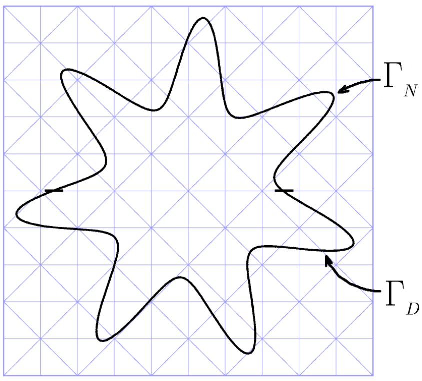
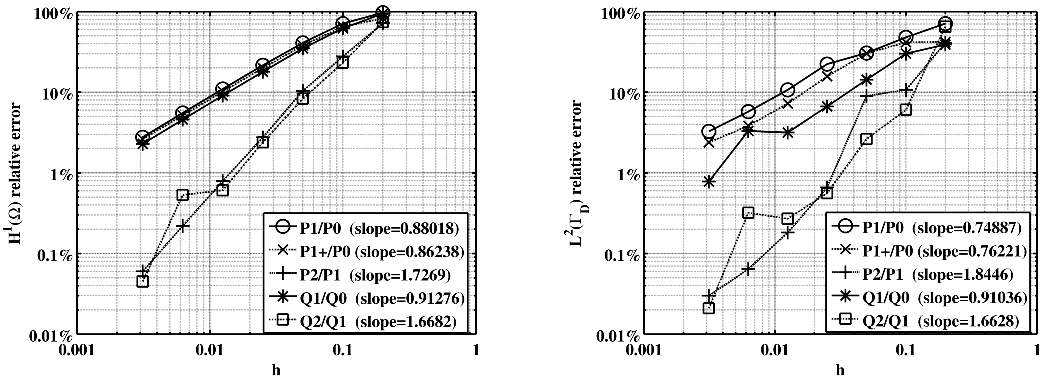


Figure 1: *The test domain on which the numerical experiments are performed. A Neumann condition is prescribed on with* Γ_N *and a Dirichlet one on* Γ_D.

The specificity of the Xfem is that it combines a level-set representation of the geometry of the crack with an enrichment of a finite element space by singular and discontinuous functions. The originality of Xfem consists in a particular way of defining the enrichment via the multiplication by a partition of unity provided by basis functions of a Lagrange finite element method.

In this work we adapt the techniques of Xfem to develop a new method allowing computations in domains whose boundaries are independent of the mesh. A similar attempt was done in [5, 6]. Our goal is to develop a fully optimal method. It can be considered as a fictitious domain type method. Its advantage, compared to existing ones (see for instance [2, 3]), is its ability to easily treat complex boundary conditions. However, the elementary matrices have to be computed taking into account the geometry of the real boundary. This method can be of interest for computational domains having moving boundaries or boundaries with a complex geometry and various conditions on them (Dirichlet, Neumann, Signorini, ...).

Some numerical experiments are performed on the domain depicted in Fig. 1. On Fig. 2 some error curves are presented where u, u^h, λ and λ^h are the exact solution, the approximated solution, the Lagrange multiplier ensuring the Dirichlet condition and the approximated multiplier, respectively.



Rate of convergence $\|u - u^h\|_{1,\Omega}$. Rate of convergence $\|\lambda - \lambda^h\|_{0,\Gamma_D}$.

Figure 2: *Rates of convergence for some couples of finite element spaces (principal unknown/ multiplier for the Dirichlet condition).*

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