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ON COMBINATIONS OF ADAPTIVE FE AND MESHLESS FDM

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ABSTRACT

Effective combination of the Meshless FD (MFDM) and FE methods is being intensively developed in the recent time [1-5,10,11,12,13,14]. Following earlier Author papers on combinations of the both FE and MFD methods [8,9,10,11], considered are here further developments of a unified approach to combination of these methods.
Several possible combinations of the FE and MFD methods were examined. One approach is based on an attempt to bring the MFDM and the FEM closer to each other. Both methods may be oriented on evaluation of the vector $\mathbf{Df} = \mathbf{Q(x)}d$ composed of function and its derivatives $\mathbf{Df} = \{f \; f_x \; f_y \; f_{xx} \; f_{xy} \; f_{yy} \ldots\}$ given as a linear combination of nodal unknowns d and an approximation matrix $\mathbf{Q}(\mathbf{x})$. In the MFDM, for a given fixed point x_i of the domain, the matrix $\mathbf{Q}$ presents meshless finite difference formulas at this point. In the FEM this matrix is formed using shape functions and their derivatives.
The other possible way of unification uses a reverse approach. Thus, using moving weighted least squares approximation the MFDM formulas are expressed in terms of the FEM notation due to definition $f = \sum_i N_i d_i$ of appropriate pseudo shape functions N_i.
Moreover a combination of the both methods may be applied at the same time - in different subdomains. This concept may be done using:
1. Kinematically admissible weighting functions [12]. This way interpolation is enforced on the boundary between subdomains where different methods are applied,
2. So called ramp functions [1,5,13]. Ramp functions are FEM shape functions. They give interpolation in the FEM nodes and have zero values in the MFDM nodes.
3. Lagrange multipliers [3] and penalty function methods.
4. Collocation approach - overlapping FEM and MFDM interpolants [8-12]. This approach is generalized in the presented work by using special *"hanging nodes"*.
5. Reproducitivity conditions in transition zone [2,4,12].

Both considered methods may be also applied in a sequence, when the MFDM (MWLS based) postprocessing is applied to smoothen rough results obtained by the FEM [7,8].

The present research, done in the domain of the adaptive techniques, includes:
1. Mesh control techniques:
(i) a'posteriori error estimation: certain generalization of the ZZ estimators [6] is done here,
(ii) mesh refinement strategy
2. Mesh refinement and/or enrichment. Regeneration of meshes is dealt with here.
3. Mapping of history dependent variables from the old to the new mesh.
4. Final postprocessing of solutions on fine meshes for additional solution enhancement.

The novel methods of a'posteriori error analysis are introduced here and several mesh refinement techniques are summarized. A ZZ energy norm estimator may be used in the MFDM as well. It may be generalized. One may use the MFDM solution taking into consideration local derivatives and consistent derivatives ones as a reference one.

Computer implementation of that FE/MFD model is done in the form of the **NAFDEM** system (**N**onlinear **A**daptive **F**inite **D**ifference and **E**lement **M**ethods [12]). A number of boundary value problems, were solved. Advantages of the proposed approach is proved.

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