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STOCHASTIC PDE's AS A TOOL FOR SOLVING JOINT VELOCITY- SCALAR PDF EQUATION IN TURBULENT REACTING FLOWS: NUMERICAL ASPECTS AND VALIDATION

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ABSTRACT

In previous works [1,2], a Eulerian Monte Carlo (EMC) method was proposed in order to solve the joint probability density function (PDF) of turbulent reacting scalars and velocity field. The emphasis was put on the derivation of the method and on its connection with Lagrangian Monte Carlo methods (LMC). EMC methods are based on stochastic Eulerian fields, which evolve from prescribed stochastic partial differential equations (SPDE's) statistically equivalent to the PDF equation. By using the notion of stochastic characteristics, SPDE's equivalent to the stochastic ODE's used in LMC methods were established. The correspondence between Lagrangian and Eulerian one-point statistics led to introduce a stochastic density, and to consider statistics weighted by this stochastic density. As a result of these procedures, hyperbolic conservative SPDE's were obtained.

Numerical issues were not dealt with in [1,2], and no validation test was performed. Thus, the purpose of this work is to develop a numerical scheme for solving the SPDE's and to assess its performances. Brownian noises only appear as multiplicative sorce terms in SPDE's. This greatly simplifies the coupling between time and space discretizations. However, some issues remain.

First, the spatial discretization should preserve monotonicity. This is made difficult because of the complexity of the SPDE's characteristic system. Second, the time disctretization should be strong stability preserving (SSP). It should also allow for an implicit treatment of chemical source terms. To meet these requirements, the following solutions are proposed. For spatial discretization, we use a second order centred monotone scheme, which do not require detailed information about the characteristic system. For time discretization, we define a new weak second order Runge-Kutta scheme. It consists in stochastic extension of an implicit (IMEX) SSP Runge-Kutta scheme. This extension is based on the work by Tocino and Vigo-Aguiar.

In order to validate this numerical scheme, we consider a 1D turbulent reactive flow. The simplified Langevin (SLM) and the interaction by exchange with the mean (IEM) models are used to represent unclosed pressure and molecular effects. With these assumptions, we perform a first test in order to check the monotonicity of the numerical scheme. It consists of a Riemann problem. At initial time, the domain is split into two parts having discontinuous states. The evolution of the discontinuity is then studied. In a second test, the return to gaussianity is examined starting from a Dirac distributed velocity at inlet. Finally, we consider an auto-ignition problem. At inlet, a stoichiometric methane-air mixture at 1500K is injected, with a Gaussian distributed velocity field. We check the convergence properties of the numerical scheme.

REFERENCES

[1] V.A. Sabel'nikov and O. Soulard, "Rapidly decorrelating velocity field model as atool for solving Fokker-Planck PDF equations of turbulent reactive scalars", *Phys. Rev. E*, **72**, pp. 016301-1–016301-22, (2005).

[2] O. Soulard and V.A. Sabel'nikov, "A new Eulerian Monte Carlo method for the joint velocity-scalar PDF equations in turbulent flow", In *ECCOMAS CFD*, (2006).