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Radial basis functions for uncertainty quantification in CFD

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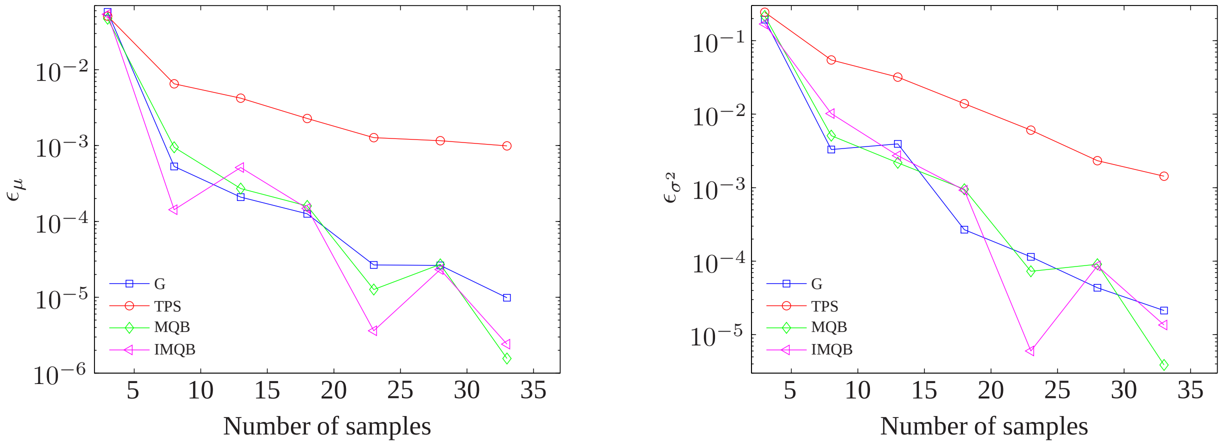
ABSTRACT

Deterministic simulations are accurate nowadays due to advanced algorithms and increasing computer power. The inherent variability of the physical system that is modeled, is often neglected. The interest in the inclusion of this variability in complex computations is increasing, since it can influence the solution significantly. Uncertainty quantification is used to compute the probability distribution of the solution based on uncertain input parameters.

When uncertainty quantification is used in combination with existing flow solvers ideally the uncertainty quantification method is non-intrusive. A non-intrusive method requires several deterministic solves using the deterministic solver as a black-box. Efficient non-intrusive methods are for example the Probabilistic Collocation method [1,2] and the Non-intrusive Polynomial Chaos method [3], which are both based on the Polynomial Chaos method [4]. For multiple uncertain parameters the amount of deterministic computations grows rapidly. For the Probabilistic Collocation method the number of points is equal to $(p+1)^d$, with p the order of the approximation and d the number of uncertain parameters. As an alternative sparse grid approaches [5] can be used to increase the efficiency. For the Non-intrusive Polynomial Chaos method the number of coefficients is $(d+p)!/d!p!$. Hosder, Walters, and Balch [3] showed that for a good Non-intrusive Polynomial Chaos approximation the amount of sampling points should be twice the number of coefficients. The polynomial chaos based methods use a global polynomial approximation of the response surface.

In this abstract the response surface is approximated using Radial Basis Functions (RBFs) through a limited number of support points. RBFs are used since they are known to be efficient interpolants in high dimensional spaces. The support points can be chosen by several sampling strategies. Here several combinations of different RBFs and sampling techniques are investigated. Recently, RBFs [6,7] became more popular for response surface approximation. Regis and Shoemaker [7] proposed a stochastic Radial Basis Function method for global optimization problems of expensive functions. They define expensive functions as function which take from minutes to several hours to evaluate. Here the focus is on CFD, where simulations take from hours to days or even weeks to compute, so the number of available support points is minimal.



(a) (b)

Figure 1: Error convergence of the different RBFs for the mean (a) and variance (b) of the mass position at $t = 10$ resulting from uncertain mass m and spring stiffness k using CVT sampling.

The convergence of the Probabilistic Radial Basis Function (PRBF) approach is numerically shown using a mass spring problem. Four commonly used RBFs are considered, i.e. Gaussian (G), Thin plate spline (TPS), Inverse multiquadric biharmonic (IMQB), and the Multiquadric biharmonic (MQB) functions. The RBFs contain a shape parameter, which is set to $c = 1$ for the convergence study. Figure 1 shows the convergence of the mean (a) and the variance (b) for the four different RBFs using centroidal voronoi tessellation (CVT) sampling, which yield an equidistant distribution of the samples. More details on the PRBF approach and the sampling of the support points can be found in reference [6].

The PRBF approach is applied to a turbulent Navier-Stokes computation around a NACA0012 airfoil with four uncertain parameters. The free stream Mach number and angle of attack are assumed to be uncertain, as well as the geometry of the airfoil. The NACA0012 airfoil is parameterized by the maximum camber, maximum camber location and relative thickness. Here the maximum camber and relative thickness are assumed to be uncertain. The free stream Mach number is 0.3 and the airfoil is at an angle of attack of 5 degrees. The deterministic simulations are performed using a commercial flow solver on a hexahedral grid of 80,000 cells. The Reynolds-averaged Navier-Stokes equations are solved using the Spalart-Allmaras turbulence model, the flow is considered fully turbulent. The free stream Mach number has a mean $\mu_M = 0.3$ and a standard deviation $\sigma_M = 0.03$, on the interval $[0.23, 0.37]$. The mean angle of attack is $\mu_\alpha = 5°$ with a standard deviation $\sigma_\alpha = 0.50°$, in the interval $[3.84°, 6.16°]$. The geometric parameters that represent the uncertainty are the thickness of the airfoil in percents of the chord with mean $\mu_t = 12\%$, a standard deviation $\sigma_t = 0.425\%$ and truncated to the interval [11%,13%] and the maximum camber in percents of the chord, which has mean $\mu_c = 0\%$, standard deviation $\sigma_c = 0.4472$ and is truncated in the interval [-1%,1%]. The uncertainty is propagated using the Probabilistic Radial Basis Function approach using the Gaussian RBF, with 35 support points obtained from Halton sampling. The flow solver is run deterministically for every sample.

Figure 2 shows the convergence of the lift-over-drag ratio with respect to the number of samples for different values of the shape parameter c using the Gaussian RBF. A larger shape parameters results in more localized RBFs, the figure shows that with this low number of samples a more global RBF results in a better approximation of the mean L/D. The mean L/D converges to 42.17, which is 1% lower than the deterministic value of 42.59. The standard deviation becomes $\sigma_{L/D} = 4.051$, which results in a coefficient of variation of $CV_{L/D} = (\mu/\sigma)_{L/D} = 9.6\%$.

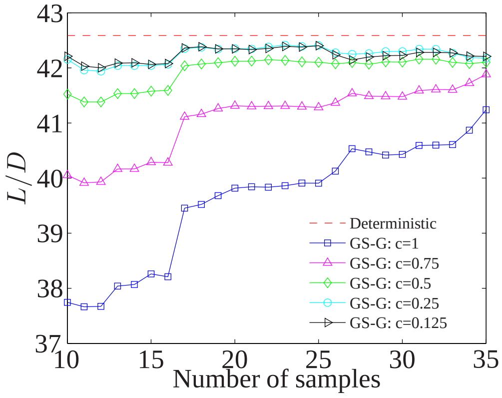


Figure 2: The convergence of the mean lift-over-drag ratio with respect to the number of samples for varying shape parameter c using the Gaussian RBF. The deterministic value is indicated by the dashed red line (--).

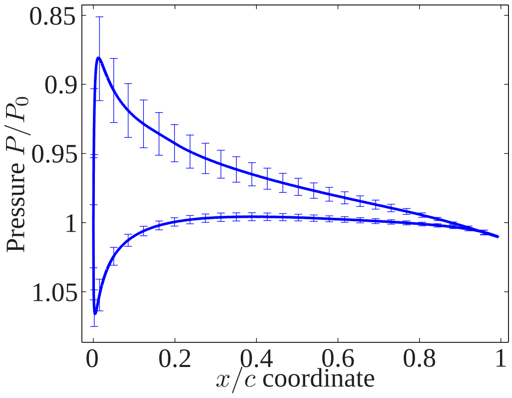


Figure 3: The mean pressure (–) along the surface of the airfoil with the bars indicating the standard deviation of the pressure, obtained using the Gaussian RBF with shape parameter $c = 0.25$.

The pressure on the airfoil surface is presented in Figure 3. The mean pressure is shown with uncertainty bars indicating the area of plus and minus one standard deviation. It can be seen that the uncertain parameters result in the largest variation in the pressure on the upper part of the airfoil and mainly near the leading edge.

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