**8th. World Congress on Computational Mechanics (WCCM8)**
**5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)**
**June 30 –July 5, 2008**
**Venice, Italy**

# ERROR ESTIMATORS FOR COULOMB FRICTION

*** Patrick Hild and Vanessa Lleras**

Laboratoire de Mathématiques de Besançon,
UMR CNRS 6623
Université de Franche-Comté,
16 route de Gray, 25030 Besançon, France.
patrick.hild@univ-fcomte.fr, vanessa.lleras@univ-fcomte.fr

**Key Words:** *Coulomb friction, a posteriori error estimates, residuals.*

## ABSTRACT

The numerical approximation of frictional contact problems occurring in structural mechanics is generally achieved using the finite element method. In order to evaluate and to control the quality of a finite element approximation, a current choice consists in developing adaptive procedures using a posteriori error estimators. The aim of the estimators is to provide the user with global and local quantities which represent in the best way the true error committed by the finite element approximation. Actually there exist various classes of error estimators, anyone showing its specificities and advantages. Some currently used estimators are e.g., those based on the residual of the equilibrium equations, the estimators linked to the smoothing of finite element stresses and the estimators based on the errors in the constitutive relation, also called ”equilibrated fluxes”.

The frictionless unilateral contact problem (or the equivalent scalar valued Signorini problem) shows a nonlinearity on the boundary corresponding to the non-penetration of the materials on the contact area which leads to a variational inequality of the first kind. For this model the residual based method was first considered and studied in [1, 5, 17] using a penalized approach. More recently the analysis without penalization term was achieved in [7], and in [8] for the corresponding mixed finite element approximation. Besides the error in the constitutive relation is considered and analyzed in [3, 15, 16] for the contact problem.

When considering friction in addition to the contact model, there are supplementary nonlinearities which have to be taken into account. The currently used friction model is the one of Coulomb (although there exist simplified and/or different models: Tresca’s friction, normal compliance, smoothed Coulomb friction... see [10, 14]) whose associated partial differential equation shows numerous mathematical difficulties which remain unsolved. In our work we consider the so-called static friction problem which roughly speaking corresponds to an incremental problem in the time discretized quasi-static model. For this model, existence of solutions hold when the friction coefficient is small enough, see [4] and the references quoted therein. When the friction coefficient is large, neither existence nor nonexistence result is available. Besides the solutions are generally non unique when the friction coefficient is large enough, see [6]. More recently a first uniqueness result has been obtained in [13] with the assumption that a ”regular” solution exists and that the friction coefficient is sufficiently small.

Our purpose in this work is to carry out a residual a posteriori error analysis for the Coulomb friction model and to obtain an error estimator with upper and lower bounds involving the discretization error (besides, note that some numerical studies dealing with error in the constitutive relation already exist for this model, see [2, 12]). To obtain the upper bound of the discretization error we consider the unique solution obtained in [13]. The complete analysis can be found in [9, 11].

This work is supported by ”l'Agence Nationale de la Recherche”, project ANR-05-JCJC-0182-01.

## References

[1] C. Carstensen, O. Scherf and P. Wriggers. “Adaptive finite elements for elastic bodies in contact”. *SIAM J. Sci. Comput.*, Vol. **20**, 1999, 1605–1626.

[2] P. Coorevits, P. Hild and M. Hjiaj. “A posteriori error control of finite element approximations for Coulomb's frictional contact”. *SIAM J. Sci. Comput.*, Vol. **23**, 1999, 976–999.

[3] P. Coorevits, P. Hild and J.-P. Pelle. “A posteriori error estimation for unilateral contact with matching and nonmatching meshes”. *Comput. Methods Appl. Mech. Engrg.*, Vol. **186**, 2000, 65–83.

[4] C. Eck, J. Jarušek and M. Krbec. *Unilateral contact problems: variational methods and existence theorems*, Pure and Applied Mathematics, CRC Press, 2005.

[5] P. Hansbo and C. Johnson. *Adaptive finite element methods for elastostatic contact problems.*, Grid generation and adaptive algorithms (Minneapolis, MN, 1997), IMA Vol. Math. Appl., Springer, New York, 113 (1999), 135–149.

[6] P. Hild. “Non-unique slipping in the Coulomb friction model in two-dimensional linear elasticity”. *Q. Jl. Mech. Appl. Math.*, Vol. **57**, 2004, 225–235.

[7] P. Hild and S. Nicaise. “A posteriori error estimations of residual type for Signorini's problem”. *Numer. Math.*, Vol. **101**, 2005, 523–549.

[8] P. Hild and S. Nicaise. “Residual a posteriori error estimators for contact problems in elasticity”. *Math. Model. Numer. Anal.*, Vol. **41**, 2007, 897–923.

[9] P. Hild and V. Lleras. “Residual error estimators for Coulomb friction”. Laboratoire de Mathématiques de Besançon, Internal report 2007-51. Submitted.

[10] N. Kikuchi and J.T. Oden. *Contact problems in elasticity*, SIAM, 1988.

[11] V. Lleras. Thesis, in preparation.

[12] F. Louf, J.-P. Combe and J.-P. Pelle. “Constitutive error estimator for the control of contact problems involving friction”. *Comput. and Structures*, Vol. **81**, 2003, 1759–1772.

[13] Y. Renard. “A uniqueness criterion for the Signorini problem with Coulomb friction”. *SIAM J. Math. Anal.*, Vol. **38**, 2006, 452–467.

[14] M. Shillor, M. Sofonea and J.J. Telega. *Models and analysis of quasistatic contact. Varational methods*. Springer, 2004.

[15] A. Weiss and B. Wohlmuth. “A posteriori error estimator and error control for contact problems”. IANS Preprint 2007/012.

[16] B. Wohlmuth. “An a posteriori error estimator for two body contact problems on non-matching meshes”. *J. Sci. Comput*, Vol. **33**, 2007, 25–45.

[17] P. Wriggers and O. Scherf. “Different a posteriori error estimators and indicators for contact problems”. *Mathl. Comput. Modelling*, Vol. **28**, 1998, 437–447.