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A LCP-BASED NUMERICAL PROCEDURE USING MESH-FREE METHOD FOR GRADIENT ELASTO-PLASTICITY CONTINUUM

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ABSTRACT

This paper presents a Linear Complementary Problem (LCP)-based numerical procedure using mesh-free method[1] for gradient elasto-plasticity continuum[2-5], in which the yield strength of strain softening materials not only depends on the effective plastic strain but also on its Laplacian. Both primary unknowns, i.e. the plastic multiplier and displacements are taken as the field variables and approximated in terms of moving least square (MLS) interpolations with their discrete values defined at nodal and quadrature points respectively. As the MLS shape function is enriched with a weight function (positive, even and with a compact support) such as the quartic spline to traditional interpolation base functions, which usually constitutes a complete basis of the subspace of polynomials, it gains an advantage over the FE shape functions in its high-order continuity particularly for the interpolation approxiamtion of the plastic multiplier in the gradient plasticity continuum[3].

The global equilibrium equation in the weak form, the non-local constitutive equation and yield criterion enforced at each local quadrature point in a strong form, i.e. in a point-wise fashion, are combined to result in a formula with the same format as that stated in the linear complementary problem (LCP[6]), i.e.

$$-\mathbf{F}^{i+1} = \mathbf{H}_A \Delta\mathbf{\Lambda}^{i+1} + \overline{\mathbf{F}}^i \geq \mathbf{0}, \qquad \Delta\mathbf{\Lambda}^{i+1} \geq \mathbf{0} \tag{1}$$

The LCP usually denoted by $(\mathbf{H}_A, \overline{\mathbf{F}}^i)$ is composed of the non-local yield criterion and non-negativity of plastic multipliers $\Delta\mathbf{\Lambda}^{i+1}$ enforced at each local quadrature point. The plastic multipliers $\Delta\mathbf{\Lambda}^{i+1}$ are then determined by using the Lexico-Lemke algorithm [6]

$$\Delta\mathbf{\Lambda}^{i+1} = LCP(\mathbf{H}_A, \overline{\mathbf{F}}^i) \tag{2}$$

A Newton-Raphson (N-R) iterative solution scheme devised for each incremental load step is derived, in which, in contrast with existing (traditional) iterative schemes, both iterations to respectively fulfill the momentum conservation and constitutive equations satisfying non-local yield criterion at each local quadrature point are simultaneously executed.

It should be stressed that as compared with the existing work[7] using the LCP setting for gradient plasticity the contributions of the proposed LCP-based numerical

procedure lie in the following points:

(1) the proposed procedure does not require to form a consistent elasto-plastic tangent matrix while the second order convergence of the iterative process is still ensured;

(2) the non-local constitutive equation and the non-local yield criterion at each local quadrature point are fulfilled in a point-wise fashion, instead of in a weak form.

Numerical results demonstrate the performance of the present numerical procedure in solving for strain localization problems. Figure 1 illustrates that though plastic strain distributions for all of three solution schemes agree well each other, but the proposed procedure performs much better than the two FE schemes in restraining spurious numerical oscillations of the axial stress around the weakened part of the bar.

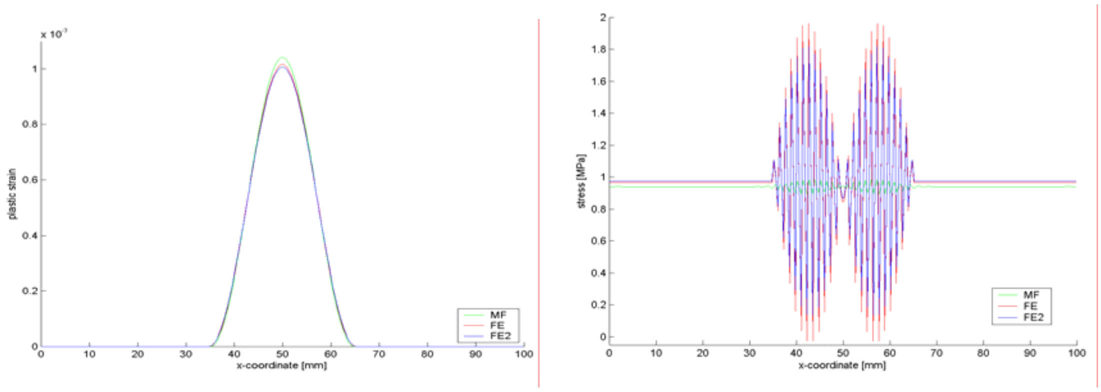

Figure 1 distributions of the plastic strain and the axial stress along the axis of the bar with strain softening material and a weakened part of central 10mm subjected to a pure tensile loading at the end of loading history (1) MF: proposed Meshfree method; (2) FE: Finite element method; (3) FE2: FE method but using the integration scheme as same as for MF.

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