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SOME REMARKS ON THE SENSITIVITY OF FINITE ELEMENTS TO MESH DISTORTIONS

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ABSTRACT

Already in the early days of the finite element method (FEM) in the sixties of the last century artificial stiffening effects, later called *locking*, and parameter dependent convergence behavior have been observed. Since the first attempts to alleviate such problems, using mixed methods, reduced integration or assumed strain methods, significant progress has been made. Throughout the decades numerous different concepts for locking-free finite elements to discretize beams, plates and shells as well as two-dimensional and three-dimensional solids have been developed.

While – mathematically – the asymptotic rate of convergence of competing elements is the same if the polynomial order of the displacement interpolation is identical, practical results may differ significantly when coarse mesh accuracy is investigated. This is particularly true when meshes are distorted. In fact, many different concepts yield identical results for rectangular elements but performance may differ significantly as elements are distorted.

Consequently, developing finite elements which are insensitive to mesh distortion has been one of the major aims of finite element developers, especially in the younger FEM-history. Apart from theoretical considerations, computational results of certain benchmark problems have established as the most widely used measure for corresponding qualities. One of the most prominent examples is the clamped beam, discretized with two four-node elements. Mesh distortion is accomplished by tilting the common edge of both elements from its original vertical alignment and distortion sensitivity is defined as the amount by which this distortion deriorates the results.

Results of benchmark problems allow for a certain judgement of finite element properties, but they are not an objective measure because only one, or several particular cases are taken into account. Moreover, additional effects may mix in and “pollute” the conclusions. In the mentioned benchmark example, alignment of element edges to the principal directions of strains and stresses is crucial; thus the *position* of the element effects the result as their *distortion* does. It is desirable, however, to have a method available which exclusively measures their distortion.

In the present study, the author tries to develop a more general and objective view of distortion sensitivity of finite elements. A method based on eigenvalue analyses of patches is proposed which measures distortion sensitivity independently, with no influence by particular boundary conditions or loads. These tests are not individual benchmark problems but cover the entire spectrum of element behavior.

Numerical results partly confirm previous findings. However, there are also some surprises. In particular, certain elements which show some distortion sensitivity in the classical beam test appear to be practically insensitive to mesh distortions. Deviations from the optimal result in the beam test are not a consequence of mesh distortions but a result of “misalignment”, as explained above.

The interrelation between satisfying the constant strain patch test (a numerical consistency test) and distortion sensitivity is also discussed. The observation that elements which have been successfully tailored to be particularly insensitive to mesh distortions fail to pass the patch test in general situations has been discussed and explained in detail by Richard MacNeal (see, for instance, his text book [1]). It can be shown that this observation is closely related to a phenomenon called *trapezoidal locking* (*curvature thickness locking* in the context of three-dimensional shell elements). It is impossible to avoid trapezoidal locking and pass the patch test at the same time. This type of distortion sensitivity thus may as well be interpreted as a “normal” locking phenomenon.

REFERENCES

[1] R. MacNeal, *Finite Elements: Their Design and Performance*, Dekker, 1993.