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Computing Elastodynamic Response Using Cellular Automata

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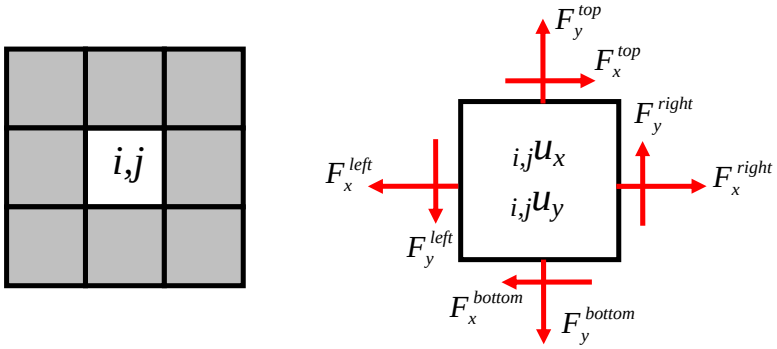
ABSTRACT

A new physics-based Cellular Automata (CA) modeling approached has been developed to compute the elastodynamic response of a two-dimensional continuum. Cellular Automata modeling has seen wide application in a diverse number of problems, particularly in the social sciences. Physics-based CA modeling followed[1], most notably in the areas of lattice gas statistical mechanics, diffusion processes, reaction-diffusion processes, phase transitions, and phenomenological models of wave propagation. The application of CA to problems in solid mechanics have been sparse, and where present, have mostly treated the continuum as cells connected to neighboring cells by elastic[2-3] or elastic and plastic[4] links. As noted by one research team[3], the Poisson effect is not modeled precisely. In addition, this approach does not generalize well to arbitrary constitutive laws.



(a) Moore neighbors (gray) of cell (*i,j*)

(b) Neighbor forces on cell (*i,j*)

Figure 1: (a) Cellular automata discretization and (b) neighbor forces

In this effort, a two-dimensional linear elastic continuum is discretized into rectangular cellular automata (see Fig. 1) and a rule set **R** is developed for evolving each cell's state based on its present state and its neighbors' states. A cell's state is comprised of its displacement and velocity components, and its external force. **R** follows from the application of Newton's Laws to the cell faces, with forces on each face arising from stresses related to strains via a linear constitutive law $\boldsymbol{\sigma} = \boldsymbol{C\varepsilon}$ expressed in Voigt

notation. Finite difference approximations are used to determine the strain $\boldsymbol{\varepsilon}$ across each of the cell's faces, where careful attention is paid to equilibrate forces on each face, and to preserve symmetry in the chosen neighbor cells (a full Moore neighborhood is used). As such, cells are linked via the constitutive relationship, vice springs or links, and thus capture correctly the material's Poisson effect. Full details can be found in (Leamy, 2008)[5].

The developed approach is advantageous in several respects and performs very well when compared to more-traditional approaches. CA computations are local and result in compact, massively parallel code. It is a 'bottom-up' approach based on local interactions that lead to emerging behavior – in this case elastodynamic waves. It is notable in that it starts and ends as a discrete formulation, avoiding the difficulties associated with development and numerical solution of partial differential equations. As such, it may easily accommodate coupled-field simulations. Results are given in Fig. 2 for an elastic half-space simulation in which a Gaussian pulse excites a point on the surface. Documented is very good agreement in the surface wave profile when compared to that from a traditional staggered-grid finite difference (FD) approach. In fact, the CA approach exhibits less ringing and better symmetry in the leftward and rightward moving waves. Other comparisons (not presented) document very good agreement in predicted shear and pressure waves.

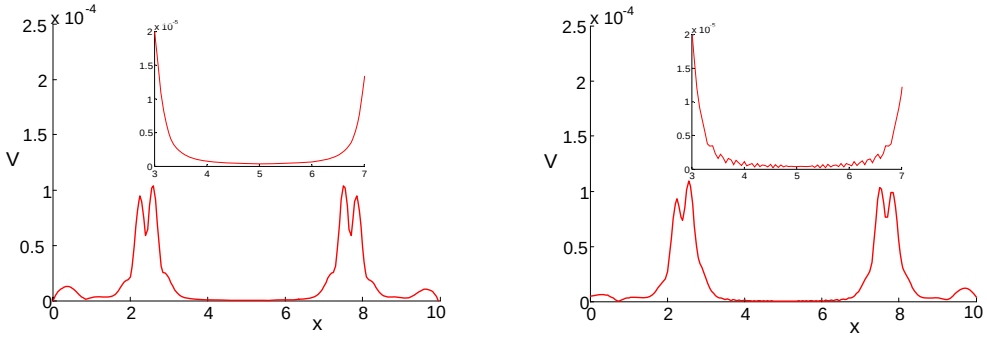


Figure 2: Simulation results comparing the developed cellular automata model to a staggered-grid finite difference model: (a) CA surface velocity magnitude, (b) FD surface velocity magnitude.

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