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A PROCEDURE FOR THE IDENTIFICATION OF CONCENTRATED DAMAGES ON BEAMS BY STATIC TESTS

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ABSTRACT

The great advantage of proposing multiple damage identification procedures has been addressed in [1] where explicit solutions have been also proposed on the basis of measurements by static tests.
In this work, by following a different strategy, an explicit solution of the static deflection function of multiple damaged beams is adopted to propose a novel procedure for the identification of both position and intensity of concentrated damages.
By making use of the generalised function theory, the closed form expression of the transversal displacement of a beam with multiple (n_λ) concentrated damages, subjected to static loads, can be written as follows:

$$u(x) = c_1 + c_2 x + c_3\left[x^2 + 2\sum_{i=1}^{n_\lambda}\lambda_i\left(x - x_i\right)U\left(x - x_i\right)\right] + c_4\left[x^3 + 6\sum_{i=1}^{n_\lambda}\lambda_i\, x_i\left(x - x_i\right)U\left(x - x_i\right)\right] + \frac{q^{[4]}(x)}{E_0 I_0} + \sum_{i=1}^{n_\lambda}\lambda_i\frac{q^{[2]}(x_i)}{E_0 I_0}\left(x - x_i\right)U\left(x - x_i\right), \tag{1}$$

in which $U(x\text{-}x_i)$ is the unit step (Heaviside) function, x_i (i=1,2,…, n_λ) are the damage positions, λ_i are the damage intensity parameters that can be associated to the crack depths, and $q^{[k]}(x)$ indicates the *k*-th primitive function of the external load $q(x)$. Eq.(1) shows that, except for the integration constants c_1, c_2, c_3, c_4 to be evaluated by enforcing the boundary conditions, the transversal displacement at abscissa x depends on the damages at positions $x_i < x$. The above mentioned property of the solution suggests, starting from the first damage, to employ two displacement measurements to recognize whether there is a damage placed at the left and activate a sequential identification procedure. For each beam interval, by making use of the analytical expression given by Eq.(1), it is possible to provide closed form expressions of the damage position and intensity as functions of the measured static transversal displacements of the beam.
As an example, let us consider a simply supported multi-damaged beam loaded by an uniformly distributed load q_o. The first two boundary conditions $u(0) = u''(0) = 0$ provide the integration constants $c_1 = c_3 = 0$. The experimental static deflection function is $u^{ex}(x)$ measured along the beam span by a non destructive test according to the measurement positions depicted in Fig.1. The identification procedure is conducted by

equating the theoretical displacements $u^{th}(x)$, given by Eq.(1), to the experimental measurements $u^{ex}(x)$, as follows:

$$u^{ex}(x_{m0}) = u^{th}(x_{m0}) \quad , \quad u^{ex}(x'_{m0}) = u^{th}(x'_{m0}) \tag{2}$$

$$u^{ex}(x_{mi}) = u^{th}(x_{mi}) \quad , \quad u^{ex}(x'_{mi}) = u^{th}(x'_{mi}) \quad . \tag{3}$$

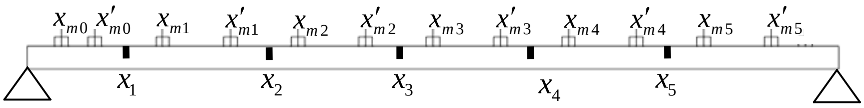

Fig.1 Scheme of the measurement positions

The solution of the system of Eqs.(2) obtained for $x_{m0} < x'_{m0} < x_1$ leads to the following explicit expressions for the identification of the integration constants c_2, c_4:

$$\begin{aligned} c_2 &= \frac{1}{x'^2_{m0} - x^2_{m0}} \left(\frac{x'^2_{m0}}{x_{m0}} u^{ex}(x_{m0}) - \frac{x^2_{m0}}{x'_{m0}} u^{ex}(x'_{m0}) - x^3_{m0} x'^2_{m0} \frac{q_o}{24E_0 I_0} + x'^3_0 x^2_0 \frac{q_o}{24E_0 I_0} \right) \\ c_4 &= \frac{1}{x'^2_{m0} - x^2_{m0}} \left(-\frac{u^{ex}(x_{m0})}{x_{m0}} + \frac{u^{ex}(x'_{m0})}{x'_{m0}} + \frac{q_o x^3_{m0}}{24E_0 I_0} - \frac{q_o x'^3_{m0}}{24E_0 I_0} \right). \end{aligned} \tag{4}$$

The system of Eqs.(3), written for couples of displacement measurements $x_{mi} < x'_{mi}$ placed between two subsequent damages, can be solved sequentially for each damage, from $i = 1$ to $i = n_\lambda$, with respect to the intensity and position as follows:

$$x_i = \frac{-a_{1i} \pm \sqrt{a^2_{1i} - 4a_{2i}a_{0i}}}{2a_{2i}} \quad , \quad \lambda_i = \frac{D(x_{mi}) + F(x_{mi})}{x_i \left[6c_4 x_{mi} + \left(\frac{q_o x_{mi}}{2E_0 I_0} - 6c_4 \right) x_i - \frac{q_o x_i^2}{2E_0 I_0} \right]} \quad , \tag{5}$$

where:

$$\begin{aligned} D(x_{mi}) &= u(x_{mi}) - c_2 x_{mi} - c_4 x^3_{mi} - \frac{q_o x^4_{mi}}{24E_0 I_0} \quad , \quad F(x_{mi}) = -\sum_{k=1}^{i-1} \lambda_k x_k (x_{mi} - x_k) \left(6c_4 + \frac{q_o x_k}{2E_0 I_0} \right) \\ a_{0i} &= 6c_4 \left[x_{mi} \left(D(x'_{mi}) + F(x'_{mi}) \right) - x'_{mi} \left(D(x_{mi}) + F(x_{mi}) \right) \right] \\ a_{1i} &= \left(\frac{q_o x_{mi}}{2E_0 I_0} - 6c_4 \right) \left(D(x'_{mi}) + F(x'_{mi}) \right) - \left(\frac{q_o x'_{mi}}{2E_0 I_0} - 6c_4 \right) \left(D(x_{mi}) + F(x_{mi}) \right) \\ a_{2i} &= -\frac{q_o}{2E_0 I_0} \left(D(x'_{mi}) + F(x'_{mi}) - D(x_{mi}) - F(x_{mi}) \right) \ . \end{aligned} \tag{6}$$

The sequential solution for the damage identification problem provided by Eqs.(5) allows to detect and quantify a damage along a chosen segment of the beam once damages lying on the left hand side have been identified. A similar solution is obtained for the case of multiple damaged beams in presence of intermediate supports.

The presented procedure represents the basis for applications to damage identification by dynamic tests.

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