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CHARACTERISTICS OF STRESS SINGULARITY FIELD OF RESIDUAL THERMAL STRESSES IN THREE-DIMENSIONAL BONDED JOINTS

***Hideo Koguchi[1] and Akira Taniguchi[2]**

[1] Nagaoka University of Technology
1603-1 Kamitomioka, Nagaoka, Niigata, Japan 940-2188
koguchi@mech.nagaokaut.ac.jp
http://www.nagaokaut.ac.jp

[2] Graduate School of Nagaoka University of Technology
1603-1 Kamitomioka, Nagaoka, Niigata, Japan 940-2188
taniguti@stn.nagaokaut.ac.jp
http://www.nagaokaut.ac.jp

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ABSTRACT

It is well known that stress singularity occurs at a vertex on the interface in joints under mechanical and thermal loadings. Stress singularity causes delamination of interface and crack occurrence near the vertex of joints, and it therefore leads to the reduction of strength in joints. In the present paper, stress distributions in stress singularity fields at the vertex on the interface in three-dimensional bonded joints are analyzed using the following BEM.

$$\boldsymbol{c}_{ij}(\boldsymbol{P})\boldsymbol{u}_j(\boldsymbol{P}) = \int_\Gamma \boldsymbol{U}^*_{ij}(\boldsymbol{P},\boldsymbol{Q})\boldsymbol{t}_j d\boldsymbol{S}(\boldsymbol{Q}) - \int_\Gamma \boldsymbol{T}^*_{ij}(\boldsymbol{P},\boldsymbol{Q})\boldsymbol{u}_j d\boldsymbol{S}(\boldsymbol{Q}) \quad (1)$$

where $\boldsymbol{U}^*_{ij}$ and $\boldsymbol{T}^*_{ij}$ are fundamental solutions for displacement and traction. Observation point, P and source point, Q, are located on the boundary of domain for analysis. t_j and u_j are traction and displacement vectors, respectively. Using the fundamental solution of Rongved's two-phase materials, mesh division on the interface is not needed.

The order of stress singularity is determined using an eigen analysis of FEM. The characteristic solution, p, has the relationship of λ=1-p with the order of stress singularity λ. p is determined from the following eigen equation in Ref. [1].

$$\left(\boldsymbol{p}^2[\boldsymbol{A}] + \boldsymbol{p}[\boldsymbol{B}] + [\boldsymbol{C}]\right)\{\boldsymbol{u}\} = 0 \quad (2)$$

where $[A]$, $[B]$ and $[C]$ are matrices composed of elastic moduli, and $\{u\}$ represents the displacement vector.

Residual thermal stresses are also determined using BEM. Generally, thermal stresses for temperature variation, ΔT, in isotropic and homogeneous materials can be expressed as

$$\sigma_{ij} = \frac{E}{1+\nu}\left(\varepsilon_{ij} + \frac{\nu}{1-2\nu}\varepsilon_{kk}\delta_{ij}\right) - \frac{E}{1-2\nu}\alpha\Delta T\delta_{ij} \qquad (3)$$

where E represents Young's modulus, ν Poisson's ratio, α represents the coefficient of thermal expansion, and δ_{ij} Kronecker's delta.

Material properties used in the analysis are shown here. Young's modulus, Poisson's ratios and a coefficient of thermal expansion are (E_1, ν_1, α_1)=(166.0GPa, 0.26,3.0×10^{-6}) for silicon and (E_2, ν_2, α_2)=(2.74GPa, 0.38, 3.3×10^{-5}) for resin. Temperature variation ΔT is -10K.

Model for analysis and its size are shown in Fig. 1. The sizes of material 1 are 0.1mm in height and 20mm in width. The thickness of bond layer, h, is 0.02mm. The minimum size of mesh near the vertex is 0.05μm.

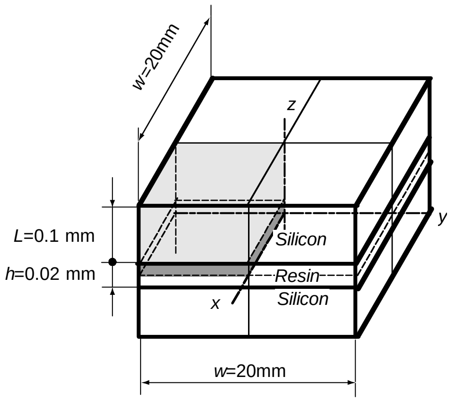
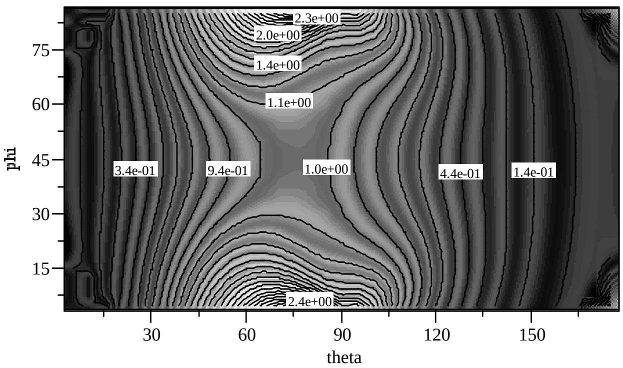
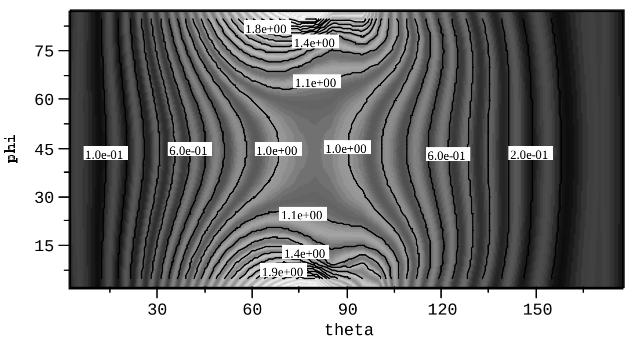


Fig.1 Model for analysis

In particular, the distribution for stress components in a spherical coordinate system with an origin at the vertex is concisely investigated. The distribution of stress, $\sigma_{\theta\theta}$, on a sphere of radius 0.1μm for BEM analysis is shown in Fig.2(a). The interface is located at θ=90deg and free surfaces are at ϕ=0 and 90deg. The distribution of $\sigma_{\theta\theta}$ calculated using eigen vector of eq.(2) is shown in Fig.2(b). It is found that these stress distributions are similar to each other. We show that stress singularity fields for the residual thermal stresses are expressed by a linear combination of angular function of θ and ϕ determined from eigen analysis. Finally, we can determine an intensity of singularity using the results of eigen analysis and BEM.



(a) BEM analysis



(b) Eigen analysis

Fig.2 Distribution of stress, $\sigma_{\theta\theta}$, on a sphere with an origin located at the vertex in a 3D-joint

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