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AN ALTERNATIVE BOUNDARY ELEMENT MULTI-REGION FORMULATION APPLIED TO 3D INFINITE DOMAIN PROBLEMS

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ABSTRACT

There are several techniques in the literature for non-homogeneous infinite domain simulation, and each one may be more advantageous than the others depending on the problem considered. In reference [1], for example, an analytical solution for a two layer infinite domain with a circular load is deduced. This solution has the advantage of being analytical, although it is only valid for a specific situation. A more general problem is considered in reference [2], in which an infinite half-space with a variable elasticity module is studied. The solution found in this case is not completely analytical, but it is valid for some different Poisson ratio and elasticity module variations. If one desires to embrace more general non-homogeneous infinite domain problems, a feasible option is to employ numerical methods such as the boundary element method (BEM). In reference [3], for instance, a 3D tunnel excavation front is modeled with the BEM.

In the literature, most of the works that employ the BEM in non-homogeneous analysis use a classic formulation, in which the regions are considered initially separated from each other and later joint together by imposing equilibrium and displacements compatibility. This approach may cause imprecision due to the conditions imposed along the contacts and leads to a large system of equations. There is, however, another technique that is more accurate than the classic one because it does not require equilibrium or compatibility equations. Hence, the system of equations is reduced because the interface tractions are not included among the unknowns. This alternative formulation was employed in reference [4] for bending plate analysis, later in [5] for bending moment calculation in plates and adapted in [6] for 3D elastic problems.

The objective of this work is to employ the alternative multi-region formulation used in references [4], [5] and [6] in a 3D non-homogeneous infinite domain problem. The expression to be considered is:

$$\left\{\sum_{s=1}^{nd}\left[\frac{E_s}{E_1}c_{ijs}\right]\right\}u_j+\sum_{e=1}^{ne}\left[\frac{E_e}{E_1}\int\limits_{\bar{\Gamma}_e}p_{ij}^{*}u_j d\bar{\Gamma}_e\right]+\sum_{c=1}^{nc}\left[\frac{\Delta E_{mn}}{E_1}\int\limits_{\Gamma_{mn}}p_{ij}^{*}u_j d\Gamma_{mn}\right]=\sum_{e=1}^{ne}\left[\int\limits_{\bar{\Gamma}_e}u_{ij1}^{*}p_j d\bar{\Gamma}_e\right] \tag{1}$$

In expression 1, the region elected to be number 1 is used to calculate the displacement fundamental solution u^*_{ij1}. The traction fundamental solution p^*_{ij} is the same for all domains because they have the same Poisson ratio. nd is the number of domains, nc is the number of contact boundaries, ne is the number of free boundaries, E_i is the Young module of region i, $\Delta E_{mn} = E_m - E_n$, c_{ijs} is a coefficient calculated for region s, u_j and p_j are, respectively, the displacement and traction at the point considered, $\bar{\Gamma}_e$ is the free boundary of region e and Γ_{mn} is the contact between regions m and n. It is important to notice that the contact tractions are not included in this expression, justifying the reduced system of equations when compared to the classic technique. Hence, for each equation written for a boundary point, all regions contributions are considered, treating the multi-region solid as a unique domain.

Expression 1 is now applied in a non-homogeneous 3D infinite domain problem, in which a half space composed by two layers is considered. The top layer has a $9000\ KN/m^2$ Young module, $0,5$ Poisson ratio and $15\ m$ of thickness, and the layer below has a $900\ KN/m^2$ Young module, $0,5$ Poisson ratio and infinite thickness. Both layers are homogeneous, isotropic, linear elastic and infinite in radial directions. A vertical circular $2\ KN/m^2$ uniform load with a $7,5\ m$ diameter is applied to the top layer surface.

This problem was simulated using a mesh with 306 nodes and 576 elements, composed by triangular elements with linear shape functions. The same mesh was employed at the surface and at the layers contact, extending to a distance from which the displacements and tractions could be considered negligible. For the nodes at these limits, the boundary values were imposed to be zero to better simulate their far field behavior. Hence, it was not necessary to close the boundary at the limits.

Considering the central point of the loaded area, a $2,5033 \times 10^{-3}\ m$ vertical displacement was obtained at the simulation. Using the values considered in this example and applying the solution given in reference [1], a $2,5000 \times 10^{-3}\ m$ displacement is obtained for this same point. The error calculated is of $0,1\ \%$, from which it may be concluded that the alternative technique presented in reference [4] leads to accurate results in infinite domain problems. One disadvantage of this technique is that it can not be used in problems in which the domains have different Poisson ratio.

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