

## Grid-Aided Stochastic Finite Element Using Ninf-G

\* Yohei Sato<sup>1</sup> and Hiroshi Okuda<sup>2</sup>

<sup>1</sup> The University of Tokyo  
5-1-5 Kashiwano-ha,  
Kashiwa, Chiba 277-8568,  
JAPAN  
sato@nihonbashi.race.u-  
tokyo.ac.jp,  
http://nihonbashi.race.u-  
tokyo.ac.jp/

<sup>2</sup> RACE, The University of  
Tokyo  
5-1-5 Kashiwano-ha,  
Kashiwa, Chiba 277-8568,  
JAPAN  
okuda@nihonbashi.race.u-  
tokyo.ac.jp,  
http://nihonbashi.race.u-  
tokyo.ac.jp/

**Key Words:** *Stochastic FEM, Recycling Krylov Method, GridRPC*

### ABSTRACT

Lately, thanks to the improvements in computer hardware and parallel algorithms, FEM models have become quite large and complex. In such situations, due to the lack of information, one needs to handle uncertainty in the objects subjected to analysis, in the conditions the analysis is performed, or in the applied mathematical models.

When variables involved in FEM possess stochastic terms, Stochastic FEM can be used to solve the problem [1][2]. In Perturbation SFEM, one needs to solve at least as many linear systems  $Ax = b$  as the number of stochastic terms. If each element has one stochastic term in its material properties, the number of systems will be the number of elements. These systems have the same coefficient matrix  $A$  and different right-hand sides  $b$ . Thus, SFEM has been considered to be unpractical for large-scale problems.

For problems involving a number of linear systems with the same coefficient matrix  $A$ , variations of modified CG solver have been developed[3][4][5][6]. In one of the methods called recycling Krylov method or seed method, the Krylov subspace generated from the seed system is reused considering the orthogonal projections in subsequent systems. This method can be applied to SFEM and reduce the computational cost of CG iterations in the calculations of the perturbation terms.

On the other hand, the aforementioned linear systems can be solved asynchronously as a task parallel problem. Because task parallel problems are very suitable for distributed environment, SFEM computations can be done efficiently on a distributed environment.

We modified and applied the seed method to SFEM in a grid friendly way in which the communication cost is low and keeps its task parallel nature. Figure 1 shows the data flow and the calculation sequence of the proposed method. With this proposed method, the calculation time for each perturbation term could be reduced to 50%. We also integrated it into a grid application using Ninf-G[7] which is one

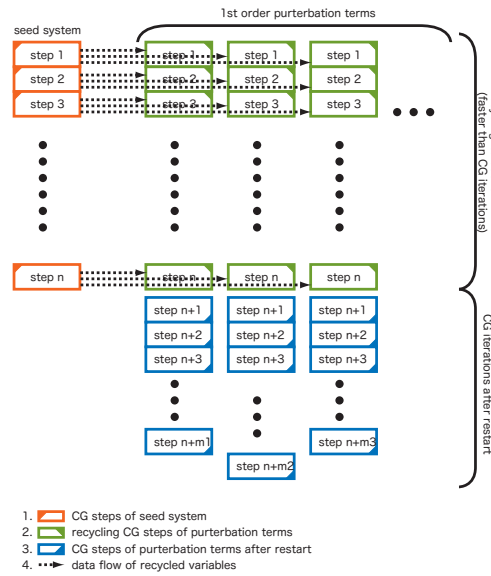


Figure 1: Data Flow and Sequence of The Proposed Method

implementation of GridRPC. Advantages of utilizing Ninf-G over our former implementation that used GT4 are easier implementation of the client application, easier handling input or output files and better fault-tolerance. The grid application has been tested on a grid test bed consisting of multiple PC clusters. This study was supported by Toray Industries, Inc and the 21st Century COE program of The University of Tokyo.

## REFERENCES

- [1] R. Ghanem and P. Spanos. *Stochastic Finite Elements: A Spectral Approach*. Springer Verlag, 1991. (reissued by Dover Publications, 2004.).
- [2] S. Nakagiri and T. Hisada. Stochastic finite element method developed for structural safety and reliability". *Proceedings of the 3rd International Conference on Structural Safety and Reliability*, pp. 395–408, 1981.
- [3] Michael L. Parks, Eric de Sturler, Greg Mackey, Duane D. Johnson, and Spandan Maiti. Recycling krylov subspaces for sequences of linear systems. *SIAM J. Sci. Comput.*, Vol. 28, No. 5, pp. 1651–1674, 2006.
- [4] C. F. Smith, A. F. Peterson, and R. Mitra. A conjugate gradient algorithm for the treatment of multiple incident electromagnetic fields. *IEEE Transactions on Antennas and Propagation*, Vol. 37, pp. 1490–1493, November 1989.
- [5] J. Erhel and F. Guyomarc'h. An augmented conjugate gradient. *INRIA*, Vol. 3287, , 1997.
- [6] D.P.O'Leary. The block conjugate gradient algorithm and related methods. *Linear Algebra and its Applications*, Vol. 29, pp. 293–322, 1980.
- [7] Yoshio Tanaka, Hidemoto Nakada, Satoshi Sekiguchi, Toyotaro Suzumura, and Satoshi Matsuoka. Ninf-g: A reference implementation of RPC-based programming middleware for grid computing. *Journal of Grid Computing*, Vol. 1, No. 1, pp. 274–278, 2003.