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A preliminary comparative study between two edge-based finite volume formulations for the solution of elliptic problems in heterogeneous and anisotropic media

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ABSTRACT

The modeling and simulation of elliptic type problems in heterogeneous and anisotropic media may represent a great challenge from mathematical and numerical point of views. Particularly, the simulation of fluid flow in non-homogeneous and non-isotropic porous media, which is a common situation associated to contaminant transport in aquifers and oil recovery in petroleum reservoirs, involves the numerical solution of an elliptic (diffusive) type equation in which the full tensor diffusion coefficient is, in general, discontinuous and may vary orders of magnitude within very short distances.

In the present work, we present a preliminary comparative study between two vertex-centered edge-based finite volume formulations (EBFV) with median dual control volumes. In the first one, which is derived from the Crumpton's two step approach, gradients are computed in a first loop over the edges of the primal mesh, and then, these gradients are used to compute elliptic terms in a second loop. This procedure produces very accurate approximations for fluxes at a cost of using a non-local stencil and solving a non-symmetric system of equations. The second approach is close related to the more traditional control volume finite element method (CVFEM), in which pressure are assumed to vary linearly over the edges, and fluxes are assumed to be constant over the elements of the primal mesh, therefore, in this formulation, while the approximation for the pressure field is formally second order accurate, fluxes are at most only first order accurate. For both methodologies,

physical media properties, such as absolute permeability or thermal conductivity are associated to the elements of the primal mesh; therefore, a control volume that comprises parts of different elements of the primal mesh can contain different permeability/conductivity values.

The two alternative formulations are capable of handling structured or unstructured meshes, and non-homogeneous and anisotropic media, even though, both are linear preserving only in general triangular meshes (simplexes) or orthogonal quadrilateral ones. The edge-based data structure has been chosen to be far computationally more efficient than their element-based counterparts.

Computational performance, accuracy and convergence behaviour will be addressed for these two edge-based formulations when solving elliptic equation with full tensor and with discontinuous coefficients. Some 2-D benchmark problems will be solved to perform the comparisons between the two formulations. Solvers and pre-conditioners implemented in Petsc (Portable, Extensible Toolkit for Scientific Computation) will be used to solve the linear system of equations. Both formulations are easily extended for the simulation of 3-D elliptic problems and results will be presented in a forthcoming paper.

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