

# ON FINITE ELEMENT METHODS WITH EMBEDDED DISCONTINUITIES FOR EVOLVING INTERFACE PROBLEMS

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**Key Words:** *embedded interface, stabilization, finite element.*

## ABSTRACT

We present recent progress in finite element methodologies for evolving interface problems. Specifically, we focus on methods that embed the interface geometry within an underlying finite element mesh. Such approaches considerably alleviate remeshing procedures and have facilitated simulations of crack growth, multiphase flow, and fluid-structure interaction, just to name a few applications.

Our approach borrows ideas from the extended finite element community, wherein enrichment with the Heaviside function is used to capture interfacial discontinuities. The pertinent issue then becomes how best to enforce interfacial constraints and accurately evaluate the interfacial velocity [1]. A number of alternative methods have been proposed for weakly enforcing interfacial constraints, including penalty methods, modified Lagrange multipliers [2], Nitsche's method, and bubble stabilization [3]. In this work, we present an efficient, stabilized method for enforcing the constraints. Rather than relying on bubbles, we employ numerical analysis to identify stabilization parameters. In some cases, the resulting element level expressions simplify to algebraic relationships that are easily evaluated. We then examine approaches for accurate evaluations of the interfacial velocity field, a critical step for coupling with level-set methods. Applications to multiphase flow, fluid-structure interaction, and modeling grain-boundary slip in polycrystalline materials are presented and discussed.

## REFERENCES

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