8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

A consistent Eulerian rate model for finite pseudoelasticity

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Key Words: *shape memory alloys, pseudoelasticity, finite deformations.*

ABSTRACT

Maximum total strains of pseudoelastic polycrystalline NiTi shape memory alloys (SMAs) are in the range of 7 % (for basic properties of SMAs see e.g. [1]). Since this magnitude might be regarded as moderate and since fundamental effects of the material might be preferably in the focus of the material modeling, common material models for SMAs are formulated within a framework of small deformations (see e.g. [2, 3, 4]). However, in so doing, it is disregarded, that a local strain of 7 % may lead to severe structural deformations. Stents for example, a common medical application for pseudoelastic SMAs, may be deformed by a magnitude of their diameter without exceeding a local strain of 7 %. Thus, to account for the large displacements, a theory at finite deformations has to be consistently derived for a realistic simulation of SMA structures.

Recently, Xiao et al. (see [5]) presented a consistent Eulerian rate formulation of finite elasto-plasticity. It is principally based on the Kirchhoff stress (weighted Cauchy stress), the stretching tensor, and the logarithmic rate, connecting the stretching tensor with the Hencky strain. The use of deformation-like variables such as elastic or inelastic strains was avoided. This framework is adopted here for the description of the pseudoelastic material behavior of polycrystalline NiTi SMAs.

The proposed material model is based on phase specific, thermo-elastic Helmholtz free energy functions of the phases martensite and austenite which are reformulated in terms of the intrinsic stresses and the temperature. Averaging both energies by the mass fraction of martensite, and including an additional energy term accounting for the interaction of the two phases (see e.g. [6]) leads to a total non-convex Helmholtz free energy function.

For the calculation of the intrinsic stresses a tensorial internal variable is introduced which can be interpreted as the average orientation of the martensite variants. With this internal variable at hand, the total stress of the material can be calculated by minimizing the total free energy in agreement with the principle of local equilibrium. Additionally, a kinematic relation only in terms of the rate of deformation can be derived, regarding the single phases as purely thermoelastic.

The two internal variables, the tensorial average orientation of the martensite variants as well as the scalar mass fraction of martensite, are quantified by evolution equations. The latter is described by a kinetic law which follows the methodology proposed by Raniecki & Lexcellent (see [2]). Regarding the orientation of the martensite it is well known on the one hand, that the average orientation follows the

current loading direction. On the other hand, it may be inferred from bi-axial experimental tests, that the angle between the average orientation of the martensite and the loading direction is restricted by a maximum value. Both observations are used to derive a respective evolution equation.

The material model is validated on experimental multi-axial tests of polycrystalline NiTi SMAs performed by Grabe & Bruhns (see e.g. [7]).

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