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A HYBRID PUFEM-MESHFREE APPROACH FOR THE SOLUTION OF PLANE STRESS PROBLEMS

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ABSTRACT

Since the mid of 1990's a lot of work has been done around the concept of the Partition of Unity Method (PUM), leading to a great number of variant of this method, above all specialized on the treatment of local discontinuities or local nonlinearities. In effects, the combination of the PUM method with the Finite Element Method (FEM) led to the so-called PUFEM, offering a concrete opportunity for the solution accuracy improvement with respect to the standard FEM analysis.

In the first and significant contributions to this method ([1], [2], [3]) the attention of the Authors has been pointed on the linear dependence (LD) problem, which arises when we try to enrich the approximating function by means of higher order polynomial functions. The LD substantially affect the solution process by introducing stiffness matrix singularities at element level and hence at global level. In order to alleviate this problem, Strouboulis et al. [5] suggested the recourse to special solver, while later Tian et al. [6] showed in their work some simple remedies to avoid this problem in some particular situations.

The aim of the Authors is to develop an efficient four-noded element starting from the basic definition of PUFEM, where the basic interpolation is enriched by introducing a set of new generalized DOFs:

$$\mathbf{u}(\mathbf{x}) = \sum_{i=1}^{n} \varphi_i(\mathbf{x}) \cdot \sum_{j=1}^{m_i} a_{ij} \psi_{ij}(\mathbf{x}) \qquad (1)$$

where n is the number of functions used for the basic interpolation and m_i is the number of the new generalized parameters introduced in correspondence of the node i. The new element is based on the standard four bilinear functions, typical for the Q4 plane stress element, while the enrichment process has been done using higher order polynomials by using the following scheme:

$$\mathbf{u}(\mathbf{x}) = \sum_{i=1}^{4} \mathbf{N}_i^k(\mathbf{x}) \cdot \sum_{j=1}^{m_i} a_{ij} \mathbf{\Psi}_{ij}^k(\mathbf{x})$$
$$\mathbf{\Psi}_{ij}^k(\mathrm{x}) = [(x^{k+h} - x_i^{k+h}) \quad ....], \quad h = 1,3,5..... \qquad (2)$$

The adopted scheme follows the one suggested by Taylor and Zienckiewicz [4], with the only difference that many terms have been eliminated from the enrichment sequence in order to avoid the LD problem.

Moreover, as evidenced in [4], the application of this method in linear elasticity showed how meshes constructed with rectangular elements were unable to guarantee the necessary stability and consistency properties at the same time. In order to solve this problem, the higher order terms, included in the previous formulation, have been treated in the same manner as for the incompatible modes, i.e. including some correction terms allowing the representation of the constant stress states also for the irregular meshes. In this way a new FEM procedure, allowing an irregular enrichment inside the mesh, has been created and tested.

Moreover, the FEM code has been connected to a specific algorithm, based on a RBFs based neural network, able to produce, on the whole mesh, an error estimation in the solution, providing the points of the domain where an improvement (and hence an enrichment) of the approximation was necessary.

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