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PRESSURE BOUNDARY CONDITION FOR THE SCHUR COMPLEMENT OF COMPRESSIBLE FLOW EQUATIONS

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ABSTRACT

Discretization of the linearized, time-harmonic Navier-Stokes equations that describe the compressional wave motion [1] in a fluid leads to large systems of simultaneous linear equations for the unknown coefficients of velocity, pressure and temperature approximation. These linear systems can be difficult to solve efficiently using standard methods. Due to the strong coupling of the unknown fields, the convergence of traditional iterative methods that involve iterating between separate solution steps for velocity, pressure and temperature unknowns has been found to be problematic. On the other hand, a major challenge encountered with an alternative solution strategy based on applying Krylov subspace methods directly to the coupled system is associated with the design of efficient preconditioners.

A viable strategy for efficient preconditioning of coupled systems is to use block triangular preconditioners derived from the Schur complement reduction [2]. While the key ideas underlying the general strategy are rather simple, often practical difficulties are however encountered in designing a preconditioner for the associated Schur complement problem [3]. Defining suitable boundary conditions for the Schur complement preconditioner can also be problematic, and the question of how the chosen boundary conditions affect the efficiency of the preconditioner is not well understood.

We describe recent advances in the design of robust and efficient block preconditioners for the flow equations considered. Following the developments of [4], we demonstrate that accurate approximations to the Schur complement operator and consistent pressure boundary conditions for preconditioners can be derived directly from the partial differential equation model. Computations demonstrate that the application of the resulting preconditioner leads to robust iterative algorithms with convergence independent of discretization mesh size.

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