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Numerical Abacuses Method based on the equivalence between the closest point projection and a geometrical bounded problem

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ABSTRACT

Computing plastic strain is a crucial issue in finite element methods. This problem is also known as closest point projection. The radial return used for circular models reduces the computations to literal expressions (Krieg and Key, 1976). But in geomechanics, the deviatoric shape of yield functions is generally non circular, like Lade (1977) or Matsuoka-Nakai (1974). Return mapping algorithm becomes cumbersome and time consuming, and many researchs are done to increase its efficiency.

Works that will be presented rather focus on a geometric based methods. It will be demonstrated that the problem of closest point projection of the trial stress on the yield surface is equivalent to a geometric one. Whereas this property is intuitive, the tools ensuring a straightforward equivalence between the two problems were to be developed.

We can use the Lode angle θ, to define polar coordinates in the deviatoric plane. Those properties of the deviatoric plane were used by Zienkiewicz and Pande (1975) who reduced a yield surface to its polar expression, and studied the shape function $g_p(\theta)$:

$$\sqrt{J_2} = \sigma^+ g_p(\theta) \qquad \left(J_2 = \frac{1}{2}\mathrm{Tr}\left(\underline{\underline{s}}^2\right) \text{ with } \underline{\underline{s}} = \underline{\underline{\sigma}} - \sigma_m \underline{\underline{\mathbb{1}}} \right) \tag{1}$$

But Zienkiewicz and Pande focused on the properties of the yield surface, not the closest point projection. For the latter problem, one lacks a proper local base : as the deviatoric stress defines a radial tensor, but is not enough for non circular criteria. So, we introduce the orthoradial tensor $\underline{\underline{v}}$, in order to define an orthogonal base on the stress space :

$$\underline{\underline{v}} = 3\frac{\sqrt{3}}{2}\frac{1}{J_2}\underline{\underline{s}}^2 - \sqrt{3}\underline{\underline{\mathbb{1}}} - \frac{9\sqrt{3}J_3}{4{J_2}^2}\underline{\underline{s}} \tag{2}$$

Then we can scale the trial stress the same way Zienkiewicz and Pande normalized the criterion, to define the geometric problem associated to the closest point projection (figure 1). The geometric problem is independent from the mechanical one, and can be solved with trigonometric and geometric laws.

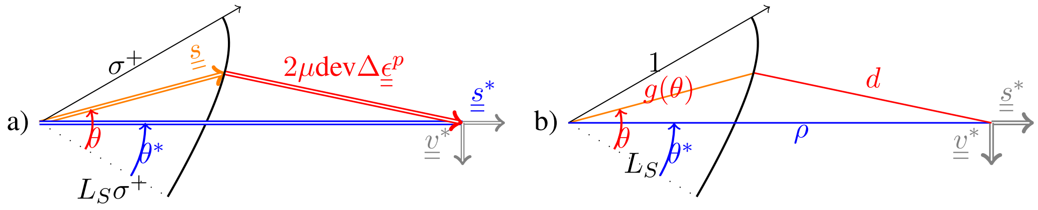

Figure 1: Equivalence between a) physical problem b) geometric

ρ_{max}	reals(thousands)	Ko	Mo
6	101	786	0,79
12	211	1644	1,61

Table 1: Size of the numerical abacuses.

As the geometric problem is a bounded, it is easyto realize numerical abacuses: for different values of (ρ, θ^*), solutions are computed and solutions are saved. The storage size is ridiculous(table 1), and the abacuses can be loaded by a FEM code, reducing computing costs. One can use explicit shape function $g_p(\theta)$ like William and Warnke (1975) or Bigoni and Piccolroaz (2004), otherwise implicit form can be deduced from criteria. Bigoni and Piccolroaz function can be used as an explicit shape function of the Matsuoka-Nakai criterion corresponding with a Coulomb (Maïolino, 2005).

The method was implemented, taking into account dilatancy and hardening, to simulate excavations in clay rocks, using a smooth version of the Hœk-Brown criterion (Maïolino, 2006).

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