Description of a composite with weak filling material by model with voids

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ABSTRACT

The goal of the presentation is to show the behaviour of a two-phase material which stands for a cellural composite with stiff skeleton and weak filling material. The filling material is modelled using Gurson-Tvergaard constitutive model.

Problem statement: The finite elements discretized incremental equation of equilibrium at time *t* with already imposed displacement boundary conditions is of the form [1, 2, 3]:

$$\left(\int_{\Omega^t} \mathbf{B}_L'^{\mathrm{T}}\, {}^t_t\bar{\boldsymbol{\tau}}\, \mathbf{B}_L'\, d\Omega^t\right)\Delta\mathbf{q} + \int_{\Omega^t} \mathbf{B}_L^{\mathrm{T}} \Delta\mathbf{S}\, d\Omega^t = \int_{\Omega^t} \mathbf{N}^{\mathrm{T}} \Delta\mathbf{f}\, d\Omega^t + \int_{\partial\Omega_\sigma^t} \mathbf{N}^{\mathrm{T}} \Delta\mathbf{t}\, d\left(\partial\Omega_\sigma^t\right)$$

where $\mathbf{B}_L'$ is the non-linear operator, $\mathbf{B}_L$ is the linear operator, ${}^t_t\bar{\boldsymbol{\tau}}$ is the Cauchy stress matrix, $\Delta\mathbf{q}$ is the displacement increment, $\Delta\mathbf{S}$ is the stress increment, $\mathbf{N}$ is a set of the shape functions, $\Delta\mathbf{f}$ is the increment of the body forces and $\Delta\mathbf{t}$ is the increment of the tractions. The integration is done over the body Ω and its boundary $\partial\Omega$ (in particular, stress boundary σ).

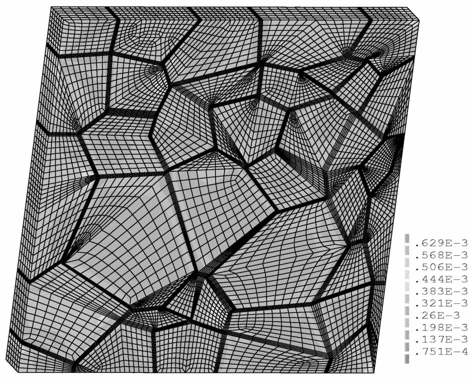
Constitutive model: This is the Gurson Tvergaard model [4, 5] with the yield function as follows

$$F = \left(\frac{\sigma^M}{\bar{\sigma}}\right)^2 + 2q_1 f \cosh\left(\frac{3q_2\sigma_m}{2\bar{\sigma}}\right) - \left(1 + q_3 f^2\right)$$

where σ^M is the Mises stress, σ_m is the mean stress, $\bar{\sigma}$ is the Mises stress in the matrix, *f* is the void ratio and $q_1, q_2\ q_3$ are the Tvergaard coefficients.

Finite strains: The gradient $\mathbf{F} = \partial(\mathbf{X} + \mathbf{u})/\partial\mathbf{x}$ is decomposed into its elastic and plastic parts, $\mathbf{F} = \mathbf{F}^e\mathbf{F}^p$. The deformation increment $\Delta\mathbf{D}$ is rotated to the un-rotated configuration by means of rotation matrix obtained from polar decomposition $\mathbf{F} = \mathbf{VR} = \mathbf{RU}$, $\Delta\mathbf{d} = \mathbf{R}_{n+1}^{T}\Delta\mathbf{D}\mathbf{R}_{n+1}$, then the radial return is performed and stresses are transformed to the Cauchy stresses at $n+1$, $\boldsymbol{\sigma}_{n+1} = \mathbf{R}_{n+1}\boldsymbol{\sigma}_{n+1}^{u}\mathbf{R}_{n+1}^{T}$. The stresses are integrated using the consistent tangent matrix and the integration is done in the un-rotated configuration as for small strains.

Example: We compare materials of different porosities of the fillers and of different stiffnesses of the skeletons. We consider a range of materials performing parametric studies. The results are qualitatively different for different porosities and stiffnesses of the skeleton. The sample works in tension and is loaded uni-axially.



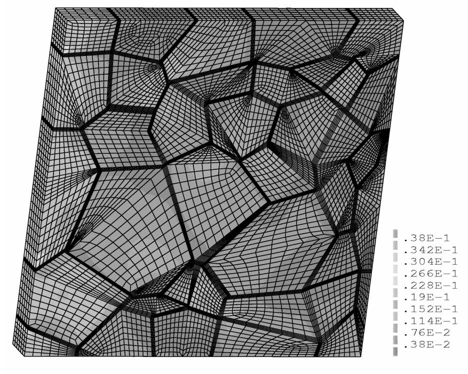


Fig. 1. The equivalent total strains distribution, strong skeleton (left), weak skeleton (right).

An exemplary results are presented in Fig. 1. We consider two cases of the material of the different Young's moduli of the filling material. The Young's moduli are 0.2E+11Pa and much lower 0.005E+11Pa. Both moduli are significantly lower than the Young's modulus of the skeleton which is 2.1E+11Pa. The yield limits are 15.0E+6Pa and 297.0E+6Pa. The initial porosity of the filler is 0.3. Observing the total equivalent strain distribution we may notice higher contrasts in the case of stronger material. The strain is distinctly lower in all interfaces than in the cells. The maximum strain, 0.691E-3 is lower than in the case of the weaker material, 0.418E-1.

Final remark: We have found that the ratio of voids and the stiffness of the skeleton changes qualitatively the behaviour of the sample. We have found that modelling of weak filling material using GT model is effective and convenient.

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