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STABILIZED FOUR-NODED ELEMENT FOR GRADIENT DAMAGE

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ABSTRACT

Introduction: One of the regularized models of quasi-brittle failure is the gradient-enhanced damage formulation [4]. To solve this coupled problem of equilibrium and nonlocal averaging, two-field finite elements are used. In three-dimensional (3D) simulations, to reduce the computational cost and avoid locking phenomena, it would be advantageous to use 8-noded elements with linear interpolation of both the displacement vector field and the averaged strain field, and one-point Gaussian integration. This however requires hourglass control of the standard continuum element, see e.g. [1], and also the stabilization of the averaging equation. Before attacking the 3D problem, this paper covers the results obtained for the 2D four-noded gradient damage element with reduced integration.

Stabilization of underintegrated gradient damage equations: One of the ways to derive the hourglass control for nonlinear models is to depart from the variational equilibrium equation according to the Galerkin method and augment it with a least square term, cf. for instance [2, 3]. This term involves a scaling factor related to the element size and stiffness modulus.

A similar approach to the averaging Helmholtz equation for the averaged strain measure, either by the Galerkin/least-square or Galerkin-gradient/least-square concept, is unsuccessful. It turns out that the stabilizing terms can be constructed using the $\boldsymbol{\gamma}$ operator as is proposed for the Laplace equation in [1]. This operator depends only on element geometry and does not influence the linear fields, but must be scaled by an arbitrary constant as well.

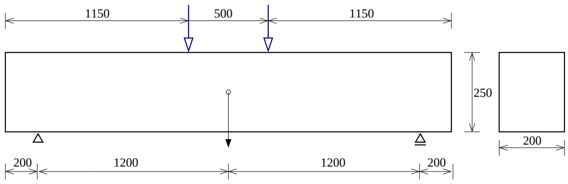
The matrix equation for the stabilized gradient damage element has the following form:

$$\begin{bmatrix} \boldsymbol{K}_{aa} + \bar{\boldsymbol{K}}_{aa} & \boldsymbol{K}_{ae} + \bar{\boldsymbol{K}}_{ae} \\ \boldsymbol{K}_{ea} & \boldsymbol{K}_{ee} + \bar{\boldsymbol{K}}_{ee} \end{bmatrix} \begin{bmatrix} \mathrm{d}\boldsymbol{a} \\ \mathrm{d}\boldsymbol{e} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_{\mathrm{ext}} - \boldsymbol{f}_{\mathrm{int}} - \bar{\boldsymbol{f}} \\ \boldsymbol{f}_{\epsilon} - \boldsymbol{f}_{e} - \bar{\boldsymbol{f}}_{e} \end{bmatrix}, \tag{1}$$

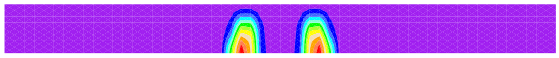
in which the stabilization submatrices are denoted by over-bars.

Numerical results: A four-point bending test of a concrete beam shown in Fig. 1a is performed. The averaged strain distributions obtained for three meshes (56×8, 112×16 and 168×24) using the stabilized finite element are presented in Fig. 1b-d. The force-deflection diagrams for the employed meshes and either full or reduced integration are compared in Fig. 2. The computation cost reduction is smaller

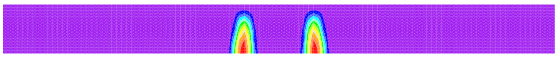
than expected, but the implementation in FEAP has not been optimized for execution time. It is expected that the gain in 3D simulations will be much larger, even though the solver efficiency is then the main computation cost factor.



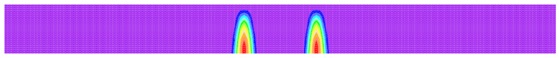
(a) Concrete beam model.



(b) Coarse mesh.



(c) Medium mesh.



(d) Fine mesh.

Figure 1: Four-point bending problem and averaged strain distributions for 3 meshes.

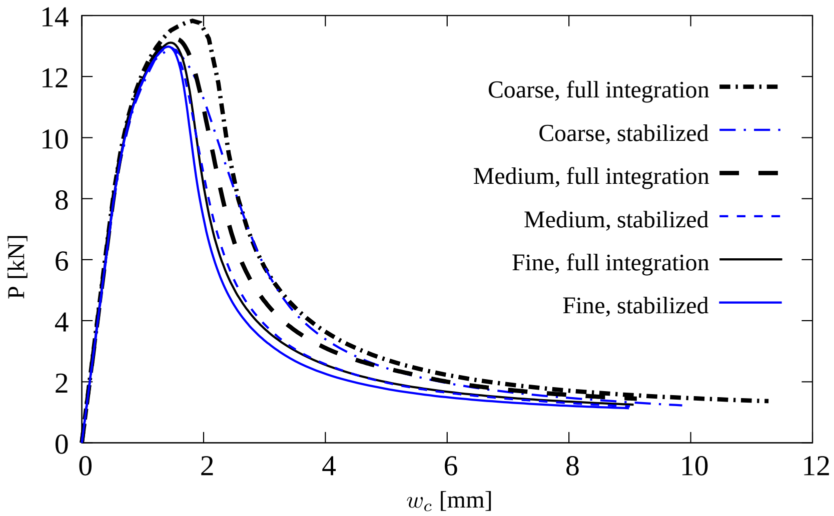


Figure 2: Force-deflection diagrams for full and reduced integration.

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