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A VARIATIONAL MULTISCALE HIGH-RESOLUTION METHOD FOR THE SIMULATION OF MISCIBLE AND IMMISCIBLE FLOW IN HETEROGENEOUS POROUS MEDIA

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ABSTRACT

Multiscale phenomena are ubiquitous to flow and transport in porous media. They manifest themselves through at least the following three facets: (1) effective parameters in the governing equations are scale dependent; (2) some features of the flow (especially sharp fronts and boundary layers) cannot be resolved on practical computational grids; and (3) dominant physical processes may be different at different scales. Numerical methods should therefore reflect the multiscale character of the solution. We concentrate on the development of simulation techniques that account for the heterogeneity present in realistic geological reservoirs, while computing the global problem on a coarse grid.

We express the governing equations of multiphase flow through porous media as a pressure equation (of near-elliptic nature) and a saturation equation (quasi-hyperbolic equation). Both are nonlinear, but only weakly coupled.

We propose a variational multiscale (VMS) method for the solution of the pressure equation, which splits the original problem rigorously into a coarse-scale problem and a subgrid-scale problem. The proposed VMS method employs a low-order mixed finite element method at the coarse scale, and a finite volume method at the subgrid scale. We identify a weak compatibility condition that allows for subgrid communication across element interfaces, which turns out to be essential for obtaining high-quality solutions. We also introduce an effective, locally conservative formulation of multiscale sources (wells) that relies on a decomposition of fine-scale source terms into coarse and deviatoric components, and on the definition of multiscale "well" basis functions.

The saturation equation is then solved on the fine scale by a high-order, unstructured grid explicit finite volume method. The fourth-order reconstruction guarantees high accuracy in the smooth regions of the flow, whereas a selective multiresolution limiter provides high-resolution of sharp fronts.

In the course of the simulation, adaptivity is used to minimize the number of local subgrid problems that need to be solved (update of multiscale basis functions) for the solution of the pressure equation.

We also use the multiscale nature of the solution to implement a logic for deciding when the pressure field must be recomputed.

We apply this numerical method to the simulation of miscible and immiscible flows in porous media. These flows are of practical importance in the areas of enhanced oil recovery by gas injection, and geological storage of carbon dioxide. The proposed approach provides accurate solutions to flow scenarios that display viscous fingering and channeling, with challenging permeability fields and concentrated fine-scale source terms.

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