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ANALYSIS OF DAMAGED VIBRATING BEAMS BY MEANS OF DISTRIBUTIONS: DIRECT AND INVERSE PROBLEMS

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ABSTRACT

The problem of damage identification by dynamic tests is of extreme importance and have been widely addressed in the literature [1]. Here a novel procedure for the identification of multiple concentrated damages on a straight beam on the basis of its eigen-mode explicit expressions is presented. The proposed method is a natural extension of the procedure presented in the companion paper [2].

By making use of the generalised function theory, a new explicit expression of the eigen-modes of a multi-cracked beam, is given as follows:

$$\phi_k(x) = c_1 f_1(x) + c_2 f_2(x) + c_3 f_3(x) + c_4 f_4(x) \quad , \tag{1}$$

with

$$\begin{gathered} f_1(x) = \sin(\beta_k x) + \sum_{i=1}^{n_\gamma} \lambda_i \mu_i H(x-x_i) U(x-x_i)\, ;\; f_2(x) = \cos(\beta_k x) + \sum_{i=1}^{n_\gamma} \lambda_i \nu_i H(x-x_i) U(x-x_i) \\ f_3(x) = \sinh(\beta_k x) + \sum_{i=1}^{n_\gamma} \lambda_i \xi_i H(x-x_i) U(x-x_i)\, ;\; f_4(x) = \cosh(\beta_k x) + \sum_{i=1}^{n_\gamma} \lambda_i \eta_i H(x-x_i) U(x-x_i) \end{gathered} \tag{2}$$

in which $U(x-x_i)$ are unit step (Heaviside) functions, β_k is the frequency parameter associated to the *k-th* natural frequency ω_k of the beam damaged at the x_i ($i = 1, 2..., n_\lambda$) positions, and $\mu_i, \nu_i, \xi_i, \eta_i$ are defined as follows:

$$\begin{gathered} \mu_i = \frac{\beta_k}{2}\left[-\sin(\beta_k x_i) + \sum_{j=1}^{i-1} \lambda_j \mu_j \hat{H}(x_i - x_j)\right];\; \nu_i = \frac{\beta_k}{2}\left[-\cos(\beta_k x_i) + \sum_{j=1}^{i-1} \lambda_j \nu_j \hat{H}(x_i - x_j)\right] \\ \xi_i = \frac{\beta_k}{2}\left[\sinh(\beta_k x_i) + \sum_{j=1}^{i-1} \lambda_j \xi_j \hat{H}(x_i - x_j)\right];\; \eta_i = \frac{\beta_k}{2}\left[\cosh(\beta x_{\gamma i}) + \sum_{j=1}^{i-1} \lambda_j \eta_j \hat{H}(x_i - x_j)\right] \end{gathered} \tag{3}$$

where

$$\begin{gathered} H(x-x_i) = \sin[\beta_k (x-x_i)] + \sinh[\beta_k (x-x_i)] \\ \hat{H}(x_i - x_j) = -\sin[\beta_k (x_i - x_j)] + \sinh[\beta_k (x_i - x_j)] \end{gathered} \tag{4}$$

and λ_i are damage parameters that can be associated to the crack depths. Eq.(1) shows that, except for the integration constants c_1, c_2, c_3, c_4 the generic eigen-mode at abscissa x depends only on the damages at positions $x_i < x$.

The particular analytical structure of the solution leads to an identification procedure which provides an explicit expression of the damage intensities as a function of the measured modes and frequencies of the beam.
As an example, the case of the beam free at its both ends depends on two constants only $c_1 = c_3 = C_k$, $c_2 = c_4 = \vartheta_k C_k$. If a non destructive dynamic test is conducted on a multi-damaged beam, the first natural frequency ω_1^{ex} and the first eigen-mode $\phi_1^{ex}(x_i)$ can be evaluated at the cracked cross-sections x_i. In this case the latter values can be equated to the theoretical expressions of the first eigen-mode $\phi_1^{th}(x_i)$ providing the following system of equations:

$$\phi_1^{ex}(x_0) = \phi_1^{th}(x_0) \quad , \quad \phi_1^{ex}(x_i) = \phi_1^{th}(x_i) \,. \tag{6}$$

Employing the first two measurements of the eigen-mode $\phi_1^{ex}(x_0)$, $\phi_1^{ex}(x_1)$ under the assumption that $x_0 < x_1$; in this case the system of Eqs.(6) leads to the evaluation of the corresponding integration constants ϑ_1, C_1 as follows:

$$\begin{aligned} \vartheta_1 &= \frac{\phi_1^{ex}(x_{m0})[\sin(\beta_1 x_{01}) + \sinh(\beta_1 x_{01})] - \phi_1^{ex}(x_{m1})[\sin(\beta_1 x_0) + \sinh(\beta_1 x_0)]}{\phi_1^{ex}(x_{m1})\left[\cos(\beta_1 x_0) + \cosh(\beta_1 x_0)\right] - \phi_1^{ex}(x_{m0})\left[\cos(\beta_1 x_{01}) + \cosh(\beta_1 x_{01})\right]} \\ C_1 &= \frac{\phi_1^{ex}(x_{m0})}{\left[\sin(\beta_1 x_{m0}) + \sinh(\beta_1 x_{m0})\right] + \vartheta_1\left[\cos(\beta_1 x_{m0}) + \cosh(\beta_1 x_{m0})\right]} \;. \end{aligned} \tag{5}$$

The further measurement $\phi_1^{th}(x_2)$ leads to an expression in which the only unknown is the first damage present at x_1. Once the first damage has been identified, the second damage intensity λ_2 can be obtained by means of the further measurement $\phi_1^{ex}(x_3)$ at x_3 and so on. The intensity λ_i of the generic damage can be written explicitly as follows:

$$\begin{aligned} \lambda_i = &\frac{[\phi_1^{ex}(x_{i+1})/C_1] - [(\sin(\beta_1 x_{i+1}) + \sinh(\beta_1 x_{i+1})) + \vartheta_1(\cos(\beta_1 x_{i+1}) + \cosh(\beta_1 x_{i+1}))]}{[(\mu_j + \xi_j) + \vartheta_1(\nu_j + \eta_j)][\sin(\beta_1(x_{i+1} - x_i)) + \sinh(\beta_1(x_{i+1} - x_i))]} + \\ &- \frac{\sum_{j=1}^{i-1} \lambda_j [(\mu_j + \xi_j) + \vartheta_1(\nu_j + \eta_j)][\sin(\beta_1(x_{i+1} - x_j)) + \sinh(\beta_1(x_{i+1} - x_j))]}{[(\mu_j + \xi_j) + \vartheta_1(\nu_j + \eta_j)][\sin(\beta_1(x_{i+1} - x_i)) + \sinh(\beta_1(x_{i+1} - x_i))]} \;, \end{aligned} \tag{7}$$

If there is no crack at the cross-section x_i, the identified damage parameter λ_i indicates the absence of damage. In the case the measurement positions are not coincident with the damage locations x_i, the measurements of the second frequency and the corresponding eigen-mode provide the sufficient additional data for the evaluation of intensity and position of a damage placed between two successive measurements.

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