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Non-smooth evolutions using the A-CD^2 method

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ABSTRACT

This paper presents a model for the description of instantaneous collisions and a computational method for the simulation of multi-particle systems' evolution. The description of the interactions among particles is required both during the regular, smooth evolution as well as in the instant of the non-smooth evolution, i.e. during a collision. The velocity-discontinuity at the instant of a collision does not allow us to solve the classical smooth equations of motion as velocities are not differentiable. In this paper, the non-smooth equations of motions describing a collision stem from the principle of virtual work.

In a multi-body system, each particle undergoes regular stress (forces) and non-regular stress (percussions). We assume that collisions are instantaneous, thus forces have to be modeled by forces concentrated in time. To compute an approximation of the evolution of the system over a time interval $[0, T]$, one has to deal with forces having a density with respect to Lebesgue's measure in time, and percussions having a density with respect to Dirac's measure in time. The A-CD^2 method consists of approximating all the forces by a succession of percussions, in order to have all the stress described as percussions. The consequence of the atomization of forces is to make the evolution a succession of instantaneous velocity discontinuities.

The interactions among particles during the collision and the non-interpenetration condition are taken into account by means of an appropriate set of constitutive laws. These two conditions are made explicit by splitting the contact percussion $\overrightarrow{P}^{int}$ into a dissipative percussion $\overrightarrow{P}^{d}$, taking into account the behavior during the collision, and a reactive percussion $\overrightarrow{P}^{reac}$ assuring the non-interpenetration of the particles. It can be shown that the internal percussion derives from a pseudopotential of dissipation and that the existence and the uniqueness of the solution is assured [2]. However, many real behaviors are well modeled by means of non associated constitutive laws. In particular, the behavior of brittle materials such as rocks, concrete or ceramics is well represented by means of Coulomb's friction law. In such a case, the problem of the existence and uniqueness of the solution can be approached by means of a series of solutions obtained by means of the Tresca associated constitutive law. It can be shown that this series converges to the unique solution of the problem if the friction coefficient verifies certain conditions [3].

The existence and the uniqueness of the solution as well as the respect of a Clausius-Duhem inequality justify the name of the proposed approach: Atomized stress Contact Dynamics respecting the Clausius-Duhem inequality (A-CD^2).

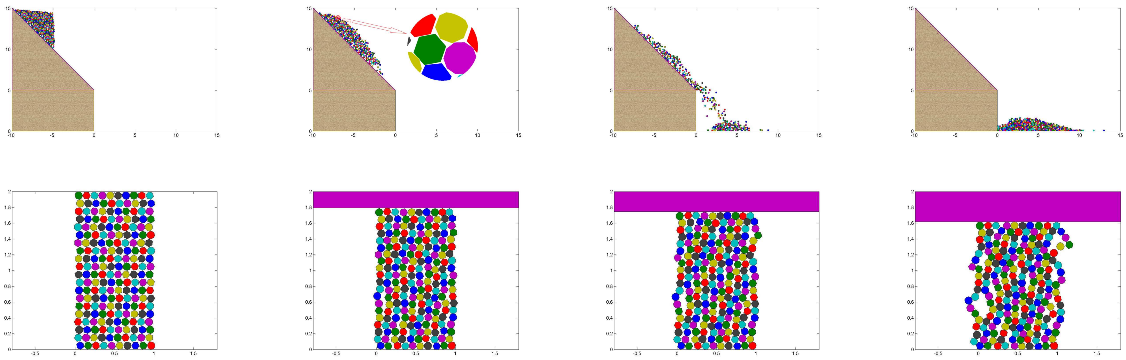

Figure 1: Evolution of the dynamic system (top) and the biaxial test (bottom)

Under the hypothesis that the percussions derive from a pseudopotential of dissipation, it can be shown that the solution of this problem is a saddle point of a Lagrangian [1]. Concerning the non-associated friction problem making use of Coulomb's constitutive law, the solution is achieved via an iterative strategy similar to the one used for demonstrating the existence and the uniqueness of the solution [3].

To illustrate the theory, some numerical simulations based on the described numerical method are finally presented. The case of an associated constitutive law has been applied to an example of a landslide formed of 300 rigid regular bodies. Similar results with more than 1200 particles can be found in [3]. In order to simulate the dispersion of material properties of an irregular assembly, a random generation of the initial configuration has been set. This means that the bodies have a random number of sides and a random initial rotation. In such a situation, the A-CD^2 method turns out to be well adapted to reproduce such a dynamic case.

The non-associated constitutive law has been applied for the simulation of a biaxial test. The geometry of the assembled specimen is given in fig.1. The assembly phase has been performed as in the previous case: the specimen is formed of an irregular packing of 200 particles, obtained by imposing a random geometrical perturbation on the initial (regular) arrangement. The last image in fig.1 shows the specimen configuration after fracture. The shear band correspond to a slipping band between particles forming an angle of about 50^o. The shear bands predicted by Mohr-Coulomb's theory ($\pi/4 \pm \phi/2$ where $\mu = tg\phi$) are therefore respected.

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