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ADAPTIVE FINITE ELEMENT METHODS FOR MULTIPHYSICS PROBLEMS

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ABSTRACT

In this talk we present an adaptive finite element method for solution of multiphysics problems and present applications involving solids, fluids, and heat transfer. We construct a multiphysics solver by connecting single physics solvers which pass information between each other. Two important and related problems are: (1) How to estimate the error in given goal quantities and (2) How to solve the resulting coupled algebraic problem efficiently.

To address (1) we assume that each individual solver is equipped with a goal oriented error estimate and an adaptive algorithm that may be used to control the error in desired goal quantities. We then construct an a posteriori error estimate for the coupled problem, which build on the a posteriori error estimates for the individual single physics solvers together with a systematic procedure for estimating the effects of the couplings. The estimates are derived using duality techniques and computable estimates are obtained using the dual weighted residual method. The strength of the couplings plays a crucial role for the propagation of error in the coupled problems as well as for iterative procedures for solution of the coupled problem and thus (1) and (2) are related. The simplest situation is that the couplings are only one-way. Then the coupled problems can be solved by solving a sequence of single physics problems. When two-way couplings are present an iterative procedure is required both for solving the primal and dual problems. The convergence of the iterative procedure is related to the strength of the couplings.

We present several realistic 3D applications including coupled flow-transport problems with applications to cooling processes[2]. The flow is governed by the Navier-Stokes equations together with a Boussinesq term to model the effects of temperature differences. We also present applications[1] involving coupled electrostatics, heat conduction, and thermoelasticity.

REFERENCES

[1] M.G. Larson and F. Bengzon, "*Adaptive Finite Element Approximation of Multiphysics Problems: A MEMS Device*", to appear in Communications in Numerical Methods in Engineering 2008..

[2] M.G. Larson, R. Söderlund, and F. Bengzon "*Adaptive Finite Element Approximation of Coupled Flow and Transport Problems with Applications in Heat Transfer"*