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ANALYSIS OF A SYMMETRIC BEM–FEM METHOD FOR THE 3D MAGNETOSTATIC PROBLEM USING SCALAR POTENTIALS

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ABSTRACT

The *magnetostatic problem* consists of finding the magnetic field caused by a stationary current source. More precisely, given a divergence–free stationary current density $\mathbf{J}$, we search for the magnetic field $\mathbf{H}$ defined in $\mathbb{R}^3$ such that

$$\mathbf{curl}\,\mathbf{H} = \mathbf{J}, \tag{1}$$

$$\mathrm{div}\,(\mu\mathbf{H}) = 0, \tag{2}$$

$$\mathbf{H}(\mathbf{x}) = O(|\mathbf{x}|^{-1}) \quad \text{when } |\mathbf{x}| \to \infty, \tag{3}$$

where μ is the magnetic permeability.

To solve this model, we can find in the literature different numerical methods; see for instance [2] and references therein. Most of these methods solve the problem in a bounded region by introducing approximated boundary conditions and applying a finite element method. These FEM techniques mainly differ in the primary unknowns of the formulation, which can be vector fields (such as a magnetic vector potential, the magnetic field $\mathbf{H}$ or the magnetic induction $\mathbf{B} = \mu\mathbf{H}$) or may be scalar potentials. According to the numerical experiments given in the literature, the scalar formulations are the most efficient both from a computational and an approximation point of view. In particular, the combination of the so called *total scalar potential* and *reduced scalar potential* seems to be specially efficient. This strategy was first introduced in [4] for 2d problems and then extended by the same authors for 3d problems; it consists in using the total potential in the magnetic materials without current sources and the reduced potential in the remaining domain. In bounded domains, this formulation means solving different Poisson problems with suitable interface and boundary conditions; recently, its numerical solution with standard finite element methods has been analyzed in [1].

In the total potential/reduced potential formulation for the problem posed in $\mathbb{R}^3$, the unknown defined outside the magnetic materials is the reduced potential and it is characterized as the solution of some homogeneous partial differential equations with constant coefficients; thus, the BEM–FEM techniques represent an interesting alternative that allow us to deal with the magnetostatic problem in $\mathbb{R}^3$ by

discretizing only the bounded domain occupied by such magnetic materials. Taking advantage of this feature, in this work we propose a symmetric BEM–FEM formulation (based on the total and reduced potentials) that covers a quite general topology for the magnetic materials and can be analyzed from a mathematical and numerical point of view by using classical results. We would like to remark that, since the magnetic domain can be not simply connected, we have to deal with a total potential which has jumps through some *cutting surfaces* and we describe an effective way to analyze this question at continuous and discrete level. Furthermore, the numerical analysis of the problem provides appropiate strategies to approximate the data given by the Biot–Savart law in order to achieve an optimal order of convergence.

We have implemented the numerical method in a MATLAB code and validated it by means of an example with known analytical solution. We have also applied the code to simulate different industrial devices. As an example, we can see in Figure 1-right the computed intensity of the magnetic field $\mathbf{H}$ obtained for an axisymmetric electromagnet with a not simply connected iron region; see a radial section of the domain in Figure 1-left. For this example, we have considered a stationary uniform current with intensity $I = 1$ A flowing through the coil, and we have taken the magnetic permeability as $\mu_0 := 4\pi 10^{-7}$ Hm^{-1} in the air and in the copper coil, and $10^4\mu_0$ in the iron.

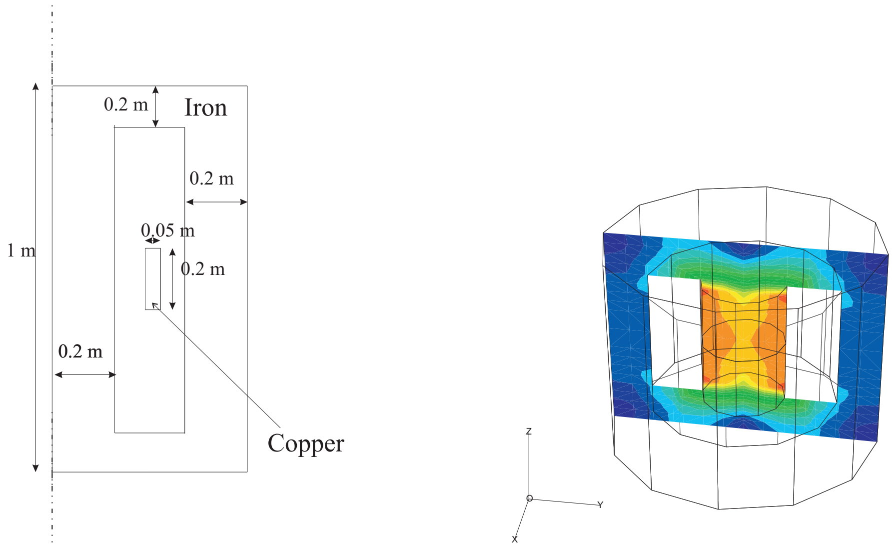

Figure 1: Radial section of the domain and computed $|\mathbf{H}|$ on a symmetry plane of the iron region.

REFERENCES

[1] A. Bermúdez, R. Rodríguez, P. Salgado. “A finite element method for the magnetostatic problem in terms of scalar potentials”. To appear in *SIAM J Numer Anal*, 2007.

[2] Ch. Magele, H. Stögner K. Preis. “Comparison of different finite element formulations for 3D magnetostatic problems”. *IEEE Trans. Magnetics*, Vol. **24**(1), 31–34, 1988.

[4] J. Simkin, C.W. Trowbridge. “On the use of the total scalar potential in the numerical solution of field problems in electromagnetics”. *Int. J. Meth. Eng.*, Vol. **14**, 423–440, 1979.