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A MOVING BODY UNDER VARIABLE FRICTION ON A TOBOGGAN

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ABSTRACT

In courses of Newtonian mechanics for engineers, the chapter on energy includes problems that conceptualizes the work of the non-conservative forces like friction. In related exercises, the friction often appears on flat surfaces but not on curves, probably for simplicity, A typical exercise in textbooks [1], is one in which a box slides from rest on a curved surface of a quarter circle (third quadrant), which ends on a flat surface; the question is about the place where box is going to stop if there is friction only on the flat surface. Now, we are just interested in knowing about the speed of the box when living the circular path taking in account that there is friction between de box and the circular surface on which it slides, furthermore the normal force is changing all time, depending on the position which makes the problem more interesting.

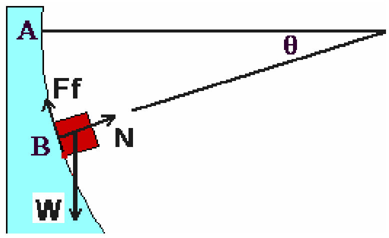

Fig. 1. A box sliding on a circular surface

The acceleration of the box (fig. 1) can be described in polar coordinates. As stated in many books of classical Mechanics [2], it is given by $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}$, where $\dot{r} = \frac{dr}{dt}$, $\ddot{r} = \frac{d^2r}{dt^2}$, $\dot{\theta} = \frac{d\theta}{dt}$, $\ddot{\theta} = \frac{d^2\theta}{dt^2}$. In this case as the path is circular of constant radius r, we have $\dot{r} = \ddot{r} = 0$, the friction force is $Ff = \mu N$ where $\mu_{kinetic} = \mu$, and the contact normal force, in this case depends on the angle $\theta(t)$. So the movement equations are:

$$N(\theta) - W\sin(\theta) = mr\dot{\theta}^2 \qquad (1)$$

$$W\cos(\theta) - \mu N(\theta) = mr\ddot{\theta} \qquad (2)$$

W is the weight of the box. By considering energy considerations including the lost of mechanical energy, in combination with equation (1) we obtain

$$N(\theta) = \frac{3}{2} m r \dot{\theta}^2 + \mu \int_0^{\theta} N(\phi) d\phi \qquad (3)$$

Which is an integral equation [3][4] that could be classified as an equation of Volterra type II. The solution of this equation requires computing methods [4].
A first attempt to solve the problem is iterating equations (2) and (3) using the Euler's method lightly modified to include the effect of the friction force which depends on the position along the path of the toboggan. In the figure 2, it is shown the iteration relations written in Mathcad (for easy writing we use ω instead of $\dot{\theta}$; α instead of $\ddot{\theta}$).

$$\begin{bmatrix} t_n \\ \theta_n \\ w_n \\ \alpha_n \\ N_n \end{bmatrix} := \begin{bmatrix} t_{n-1} + dt \\ \theta_{n-1} + w_{n-1} \cdot dt \\ w_{n-1} + \alpha_{n-1} \cdot dt \\ \frac{g}{r} \cdot \cos\left(\theta_{n-1}\right) - \frac{\mu}{m \cdot r} \cdot N_{n-1} \\ \frac{3}{2} \cdot m \cdot r \cdot \left(w_{n-1}\right)^2 + \mu \cdot N_{n-1} \cdot w_{n-1} \cdot dt \end{bmatrix}$$

Fig. 2. Iteration method

Graphs for speed given by the iteration method are similar to the one given by the simulation in Interactive Physics (fig. 3). To contrast the results, we considered the case of no friction where analytical solution is known, and it was found that the error of the iteration was 0.5%, while the error of the simulation is 1.5%. When the coefficient of friction is 0.2, the final speed is less than the case without friction. As it is observed in the figures the speed at the end of the toboggan are 3.331 m/s for the iteration method and 3.098 for the simulation, with a difference around 7% between them.

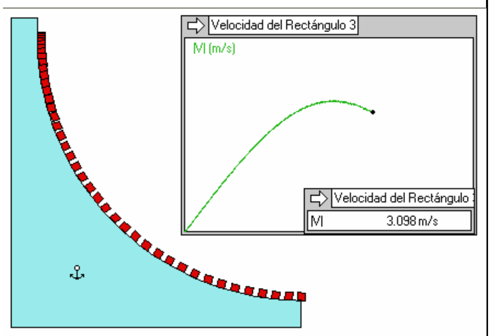

Fig. 3. Simulation in Interactive Physics

The existence of computers and computing methods has let the opportunity to solve real problems where approximations cannot take place. By studying the box on the toboggan with friction, it was possible to find a very simple numerical solution to find the final speed; the found integral equation of course may be solved by more elaborated methods. By comparing the solutions we can get more or less certainty on those programs.

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