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A new accurate yet simple shear flexible triangular plate element with linear bending strains

*** L. Damkilde**[1]

[1] Esbjerg Institute of Technology
Aalborg University
Niels Bohrs Vej 8, DK-6700
Esbjerg, Denmark
ld@aaue.dk

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ABSTRACT

Plate bending elements have been and still are the subject of many papers. Zienkiewicz, see [1], has given a very good overview of the historical development dating back to around 1965. In this paper focus will entirely be on plate elements taking the shear flexibility into account i.e. using Reissner-Mindlin plate theory. Two main approaches have been used either a displacement based approach or a hybrid interpolation approach. In this paper focus is entirely on the displacement based method. In Reissner-Mindlin plate theory the shear forces are described independently of the bending moments in opposition to the Kirchoff-Love plate theory. That means for a displacement based method both the transverse displacement w and the rotations of cross-sections of the plate θ_x and θ_y have to be interpolated separately.

A very important issue in shear flexible elements is the risk of shear locking which can lead to erroneous results. Shear locking will typically show up when the plate becomes thin and the results will not be similar to the results for an equivalent analysis using Kirchoff-Love theory. Another important issue in displacement based method is either to have fully compatible elements or at least full-filling the patch-test i.e. that the incompatibilities at the elements boundaries is sufficient small.

In the paper a new triangular plate bending element with linear bending stress is formulated. The trend in finite elements have been towards relatively simple elements, but to the authors belief many applications will benefit from a more accurate element. When the bending strains vary linearly the rotations θ_x and θ_y have to vary quadratically. For a triangular this is most easily secured by choosing two rotations in corner and midside-nodes. This will give 12 unknowns for the bending part.

The shear strains are defined by one of the cross-sectional rotations θ_x and θ_y and the derivative of the transverse displacement w with respect to either x or y. In order to have a fully consistent interpolation of the shear strains we choose a cubic interpolation which will give 10 degrees of freedom. The nodal degrees of freedom could be chosen as a transverse displacement in each corner, 2 at each side placed equidistant and 1 in the center of the element. However, we have chosen a slightly different approach

by replacing the 2 midside displacements by a transverse displacement and a rotation perpendicular to the side in the mid-side node. This formulation enables both an easier interface with traditional plate elements and elimination of a degree of freedom.

From the interpolation of the 22 degrees of freedom the stiffness matrix can be calculated using standard techniques. The transverse displacement in the center can be eliminated at element level using standard procedure. In the mid-side nodes there will be 4 degrees of freedom namely a transverse displacement, 2 rotations of the cross-sections and finally a rotation of the mid-surface. The last degree of freedom is not very practical to implement in commercial systems, but if interelement continuity is enforced the element would be fully compatible. It can be shown that if the inter-element continuity is not enforced the element will still pass the patch test. By neglecting the inter-element continuity the three mid-side rotations can be eliminated at element level leading to an element with 18 degrees of freedom.

The element has been implemented and tested on a number of cases found in the literature. The element exhibits no shear locking. In linear tests the element performs excellent and in the thin limit the results is similar to those found in [2], which describes a very accurate Kirchoff-Love plate element. In dynamic and stability tests the element perform very well and again comparable to the results from [2].

REFERENCES

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