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A C^1 finite element for three-dimensional gradient elasticity

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ABSTRACT

Gradient elasticity, as introduced by Mindlin [1], assumes the dependence of the potential energy density on both first and second gradients of the displacement field. If a usual finite element formulation is used, where only the displacement field is discretized, the presence of second gradients of displacements leads to the requirement for a smooth, C^1 interpolation of the displacements instead of just a continuous, C^0 interpolation. Some two-dimensional C^1 elements have been developed for use in plate bending problems and have been succesfully applied in problems of gradient elasticity and elastoplasticity [2,3], but no three-dimensional elements were encountered in the literature.

In this work we present a newly-developed three-dimensional C^1 element based on the two-dimensional Hermite isoparametric element proposed by Petera and Pittman [4]. The proposed element is an isoparametric 8-node hexahedron. The degrees of freedom at each node are the values of the displacement, its first derivatives, its mixed second derivatives and its fully mixed third derivative. There are therefore 64 DOFs for every displacement component, allowing for a full cubic polynomial interpolation. To pass from the element's parametric space to the cartesian space, we need the derivatives of the cartesian coordinates with respect to the parametric ones. These are obtained through a pre-processing smoothing procedure initially introduced in [4] and applied here in a modified form.

The numerical behaviour of the element has been tested extensively. The element passes successfully the single-element and patch tests, while its stiffness matrix has six zero eigenvalues as expected. Additional benchmark tests were used, where the numerical solution was compared to the one that was derived analytically. The results for the shearing and torsion of a hollow cylinder show a good convergence rate as the mesh density increases (see figure 1). Additionally it is seen that just a few elements are enough to obtain an adequate approximation of the analytical solution. This is a good indication that the increased accuracy provided by the cubic interpolation counterbalances in practical use the increased computational cost per element introduced by the larger number of DOFs.

Alternative formulations that avoid the C^1 requirement have been proposed, usually based on a mixed formulation with simultaneous discretization of both the displacement and the strain fields (see for example [5,6]). Since these have generally been used for two-dimensional elements, it is not possible

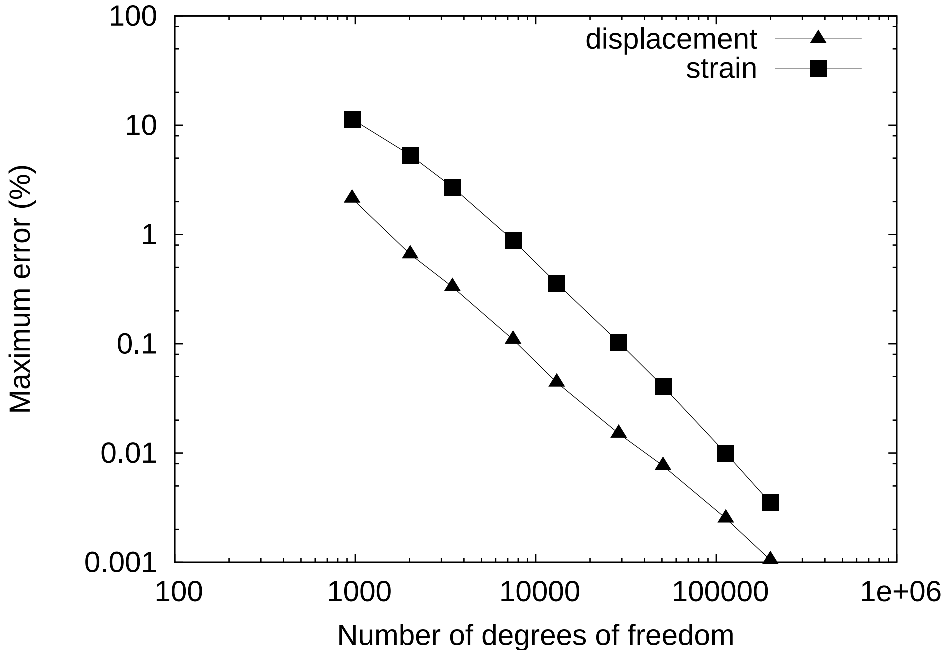

Figure 1: Results for hollow cylinder shearing benchmark, showing the convergence rate for different mesh densities.

to provide a direct comparison of the numerical behaviour for specific problems. An overview is given however, at a theoretical level, of the relative merits of the proposed formulation compared to the threee-dimensional extension of other formulations available in the literature. One advantage of the proposed formulation is that all the DOFs are used to interpolate the requested field, while in mixed formulations two fields are discretized but only one is actually used.

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