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Application of the level set X-FEM to sloshing problems of a liquid storage container

Toshio Nagashima

Sophia University
7-1 Kioicho, Chiyoda-ku,
Tokyo 102-8554, JAPAN
nagashim@me.sophia.ac.jp

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ABSTRACT

The governing equations of the flow field, including the free surface, in a rigid container using the linear potential flow theory can be discretized by the conventional finite element method (FEM), and therefore the eigenvalue and transient response analyses for the sloshing motion can be performed. In practical analyses, a liquid in a container, the geometry of which is usually designed by a 3DCAD system, can be modeled by tetrahedral finite elements, which can be generated by automatic mesh generation techniques. Since the geometry of the free surface must coincide with that of the finite element boundary in this case, the finite element model should be regenerated each time the liquid level of the container changes. Even if the volume of the liquid is constant, the finite element model requires remeshing when the configuration of the container is modified, because the liquid and the free surface have various shapes. Moreover, it is difficult to predict the geometry of the liquid and the free surface exactly in closed form for an arbitrary container, except for containers having simple shapes, such as a sphere.

Recently, in the field of computational solid mechanics, the extended finite element method (X-FEM) was proposed [1], and this method has been applied to solve stationary crack and crack propagation problems in the field of the fracture mechanics. X-FEM can express the discontinuous displacement field using a finite element interpolation function with enrichment functions, which satisfies the partition of unity condition. X-FEM can model the discontinuous displacement field near the free surface using the step function, which takes a value of 1 inside the domain and a value of 0 outside the domain. Using such a method, the geometry of the free surface can be modeled independently of the finite element mesh. The level set method [2], which is a numerical technique for tracking interfaces and shapes, has also been proposed. In this method, the geometry of the boundary surface can be described approximately by the isotropic surface with the zero value of the level set function. X-FEM analysis can be performed more efficiently in conjunction with the level set method because the geometry of the boundary surface can be implicitly described without meshing [3].

In the present study, the level set X-FEM is applied to the sloshing analysis of a rigid container with a free surface. In the proposed method [4], the entire domain of the container, which may be filled with liquid, is modeled by four-node tetrahedral finite elements and the value of the level set function, which means that the signed distance function with respect to the free surface, is evaluated for all nodes of the finite element

model. According to the value of the level set function at the four nodes of each tetrahedral element, the elements can be classified as internal elements, external elements, and boundary elements that include part of the free surface. Natural frequencies of free surface sloshing motion in rigid containers of various shape can be computed by the proposed method. **Fig.1** shows the finite element models for a cylindrical container filled with liquid of various liquid levels. The calculated sloshing natural frequencies for the several modes are shown in **Fig. 2**, as compared with the theoretical value. The calculated results obtained by the proposed method agree well with the theoretical values. It was shown that the proposed method can perform sloshing analysis efficiently for rigid containers with various liquid levels and configurations.

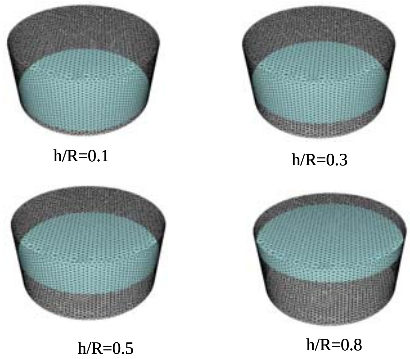

Fig.1 Finite element models for a cylindrical container filled with liquid

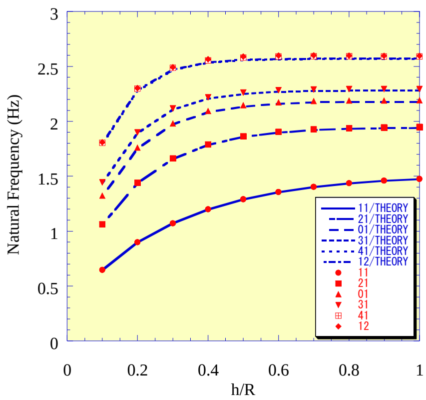

Fig.2 Sloshing frequency of a cylindrical container with liquid

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