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COMPARISON BETWEEN ε AND k BASED TRANSPORT-EQUATION SUBGRID MODELING.

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ABSTRACT

We present a comparison between subgrid modeling based on the transport-equation for the mean subgrid dissipation rate $\overline{\varepsilon}$, with that based on a transport-equation for the mean subgrid kinetic energy $\overline{K}$. First we present some theoretical arguments in favor of $\overline{\varepsilon}$-based modeling, and then we check whether or not these theoretical motivations have a substantial impact on realistic computations.

The first model tested is the, renormalization group derived, subgrid model based on $\overline{\varepsilon}$ [1]

$$\nu(\Lambda_c) = a\overline{\varepsilon}^{1/3}\Lambda_c^{-4/3}, \quad (1)$$

$$\frac{D\overline{\varepsilon}}{Dt} = \nu(\Lambda_c)\Lambda_c^2(C_{\varepsilon 1}P_K - C_{\varepsilon 2}\overline{\varepsilon}) + \frac{\partial}{\partial x_i}(\alpha\nu(\Lambda_c)\frac{\partial\overline{\varepsilon}}{\partial x_i}), \quad (2)$$

with the model constants $a = 0.46$, $\alpha = 1.39$, $C_{\varepsilon 1} = \frac{4}{3}$ and $C_{\varepsilon 2} = 2$. $D_t \equiv \partial_t + U_i\partial_i$ is the convective derivative and $P_K = -\tau_{ij}S_{ij}$ the production of turbulent kinetic energy (τ_{ij} is the subgrid stress and S_{ij} the mean strain rate tensor). $\Lambda_c = \pi/\Delta$ (with Δ the filter width) denotes the wavenumber that distinguishes between super- en subgrid scales. The second model investigated is the more standard, $\overline{K}$ based subgrid model

$$\nu(\Lambda_c) = b\overline{K}^{1/2}\Lambda_c^{-1} \quad (3)$$

$$\frac{D\overline{K}}{Dt} = P_K - c\overline{K}^{3/2}\Lambda_c + \frac{\partial}{\partial x_i}(\alpha\nu(\Lambda_c)\frac{\partial\overline{K}}{\partial x_i}), \quad (4)$$

with $b = 0.314$ and $c = 3.14$ (these constants were also obtained from the same RG calculations).

Finally, when the length scale is prescribed (by Δ in our case), one can also construct a $\overline{K}$ based model from the $\overline{\varepsilon}$ model directly through the transformation $\overline{\varepsilon} = c\overline{K}^{3/2}\Lambda_c$. Under the assumption of a constant filterwidth Λ_c, the application of this transformation to (1)-(2) results in the following alternative $\overline{K}$-based subgrid model:

$$\nu(\Lambda_c) = ac^{1/3}\overline{K}^{1/2}\Lambda_c^{-1} \quad (5)$$

$$\frac{D\overline{K}^{3/2}}{Dt} = \frac{1}{c}\nu\Lambda_c(C_{\varepsilon 1}P_K - C_{\varepsilon 2}c\overline{K}^{3/2}\Lambda_c) + \frac{\partial}{\partial x_i}(\alpha\nu\frac{\partial\overline{K}^{3/2}}{\partial x_i}). \quad (6)$$

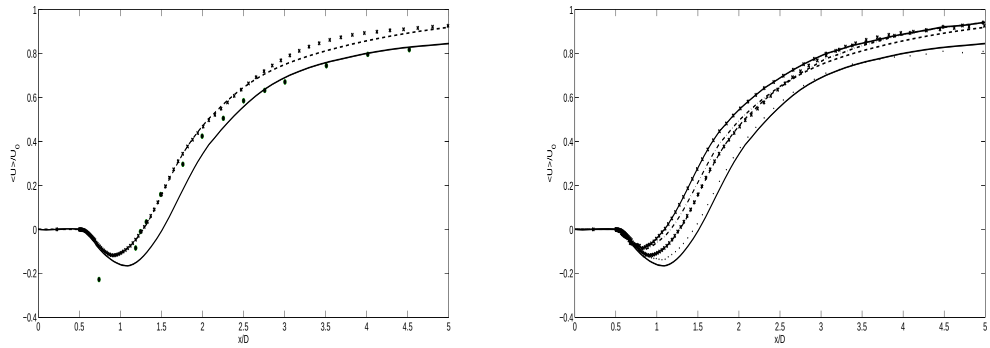

Figure 1: Left: mean centerline velocities obtained on the medium grid with the $\overline{\varepsilon}$ model $(-)$, the $\overline{K}$ model $(--)$ and the alternative $\overline{K}$ model $(\times)$, compared with experiment [(circles). Right: comparison between the medium and coarse grid results (same symbols as left figure for the medium grid results, and on the coarse grid: $\overline{\varepsilon}$ model $(\cdot)$, $\overline{K}$ model $(-\cdot)$ and the alternative $\overline{K}$ model $(-\times)$).

The equilibrium relation $\overline{\varepsilon} = c\overline{K}^{3/2}\Lambda_c$, used in the construction of both $\overline{K}$ equations, is only an approximate equilibrium assumption, supposed to be valid in the inertial range. It has been experimentally and numerically checked by several authors (see e.g. [2] and references therein), and found to be quite inaccurate. Strong deviations of the exponent from the value $3/2$ were noticed, ascribed to non-equilibrium and intermittency effects. Aside from the questionable validity of the model $\overline{\varepsilon} = c\overline{K}^{3/2}/\Delta$, the transformation leading from (1)-(2) to (5)-(6) has several other conceptual shortcomings, such as the non-uniformity of the transformation on stretched grids and the fact that one transforms from a relatively filter width-independent variable $\overline{\varepsilon}$ (which scales approximately as Δ^0 in the inertial range) to a strongly filter width dependent variable $\overline{K}$. On non-uniform grids, this again leads to extra uncertainties [1]). The goal of this work is to assess whether the above criticisms on the standard $\overline{K}$-equation, or the one derived from transforming the $\overline{\varepsilon}$-equation, have any substantial consequences in practice. We thereto applied the above three models (with appropriate near-wall modifications) to the LES of turbulent flow around a square cylinder. In Fig.1 we show the mean center line velocity obtained with the three models, on a medium and coarse grid (with respectively $1.7e6$ and $9.7e5$ grid points), compared with the experimental results of Durao e.a. [3]. On the medium all models show comparable quality, the $\overline{\varepsilon}$-based model showing somewhat better agreement in the wake at $x/D > 2$. On the coarse grid the differences between the models get larger, and one can see that the result obtained with the $\overline{\varepsilon}$ based model is less dependent on the grid resolution than both $\overline{K}$-based models. We are currently running the models on a finer grid (with $4.75e6$ grid points) to further investigate the grid-dependence of the three models.

REFERENCES

[1] C. De Langhe, B. Merci and E. Dick. “Hybrid RANS/LES modeling with an approximate renormalization group. I: model development.”. *J. Turb.*, Vol. **6**, No. 13, 2005.

[2] S. G. Chumakov. ”Scaling properties of subgrid-scale energy dissipation”. *Phys. Fluids*, Vol. **19**, 058104, 2007.

[3] D.F.G. Durao, M.V. Heitor and J.C.F. Pereira. ”Measurements of turbulent and periodic flows around a square cross-section cylinder”. *Exp. Fluids*, Vol. **6**, pp. 298-304, 1988.