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PROBABILISTIC OPTIMIZATION OF FRICTION DAMPING DEVICES

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ABSTRACT

In earthquake engineering the design of energy dissipating devices play an important role to ensure structural safety and integrity. Such devices one one hand must allow for a sufficiently high level of energy dissipation in order to reduce structural damage, and on the other hand must provide enough stiffness in order to prevent excessive permanent deformations or offsets. This leads to trade-off considerations which can be dealt with through an optimization process. The ground acceleration process $a(t)$ is assumed to be represented by an amplitude-modulated white noise process

$$a(t) = e(t)B(t) \tag{1}$$

in which $e(t) = 4 \cdot [\exp(-0.25t) - \exp(-0.5t)]$ is a deterministic modulating function and $B(t)$ is a stationary random process with given power spectral density $S_{BB}(\omega)$. In the numerical example, $S_{BB}(\omega) = const$. The structural model is assumed to have two degrees of freedom x_1 and x_2. The ground acceleration is transferred to the structure via a friction damping device with a displacement DOF x_0 and an internal plastic displacement variable z. The damping device has a mass m_0, an initial stiffness $k_{00} + k_0$, a plastic limit force s and a plastic (post-yielding) stiffness k_0. The structure is represented by two masses m_1 and m_2 as well as two elastic springs k_1 and k_2. The system state is described completely by a state vector $\mathbf{y}$ with 7 components:

$$\mathbf{y} = [z, x_0, \dot{x}_0, x_1, \dot{x}_1, x_2, \dot{x}_2]^T \tag{2}$$

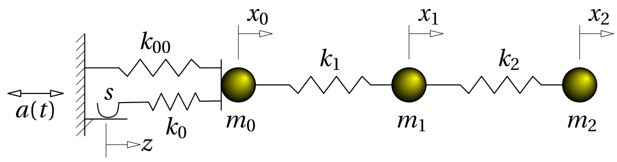

Figure 1: **Structural model with two degrees of freedom**

The equation for the derivative $\dot{z}$ of the plastic variable depends on the state of the system, primarily the magnitude of the force $F_0 = k_0(x_0 - z)$ in der spring k_0. If the absolute value of this force is smaller than the friction limit s, then the friction device responds elastically to infinitesimal state changes. This means that the friction element is blocked and $\dot{z} = 0$. If the friction limit is reached, i.e. $F_0 = s$, then the increment of z depends on the sign of

$\dot{x}_0$. If $\dot{x}_0 < 0$, the friction device returns to the elastic state and t $\dot{z} = 0$. Otherwise there is increased plastic deformation with $\dot{z} = \dot{x}_0$. For $F_0 = -s$, $\dot{x}_0 > 0$ implies return to elastic state ($\dot{z} = \dot{x}_0$) and for $\dot{x}_0 < 0$ there is increased friction slip with $\dot{z} = \dot{x}_0$.

$$\dot{y}_1 = \dot{z} \begin{cases} 0 & |k_0(x_0 - z| < s \\ 0 & [k_0(x_0 - z) = s] \wedge [\dot{x}_0 < 0] \\ 0 & [k_0(x_0 - z) = -s] \wedge [\dot{x}_0 > 0] \\ \dot{x}_0 & \text{else} \end{cases} \tag{3}$$

For the remaining state variables we have the standard equations of motion in first-order form:

$$\begin{aligned} \dot{y}_2 &= \dot{x}_0 = y_3 \\ \dot{y}_3 &= \ddot{x}_0 = -a(t) - \left[(k_00 + k_0 + k_1)y_2 - k_0 y_1 - k_1 y_4 + c_1 y_3\right]/m_0 \\ \dot{y}_4 &= \dot{x}_1 = y_5 \\ \dot{y}_5 &= \ddot{x}_1 = -a(t) - \left[(k_1 + k_2)y_4 - k_1 y_2 - k_2 y_6 + c_2 y_5\right]/m_1 \\ \dot{y}_6 &= \dot{x}_2 = y_7 \\ \dot{y}_7 &= \ddot{x}_2 = -a(t) - \left[k_2 y_6 - k_2 y_4 + c_3 y_7\right]/m_2 \end{aligned} \tag{4}$$

Here c_1, c_2, c_3 are mass-proportional damping factors.

For a fixed time step Δt this system of differential equations with given initial values for $\mathbf{y}$ can be integrated explicitly e.g. by the Euler method. Using this numerical solution scheme, the first passage probabilities of various response quantities can be easily approximated using the First-Order Reliability Method (for enhanced accuracy in conjunction with the Importance Sampling Method, a detailed discussion is given in [1]). The parameter s of the friction device is then used as design parameters in an optimization process attempting to minimize the first passage probabilities (expressed by the corresponding safety index β) of various response quantities. For the case of the first passage probability of the relative displacement $\Delta x_1 = x_1 - x_0$ the dependency of β on the friction force s is shown in Fig. 2. It is seen that there is a clearly pronounced optimal value around s = 40 kN.

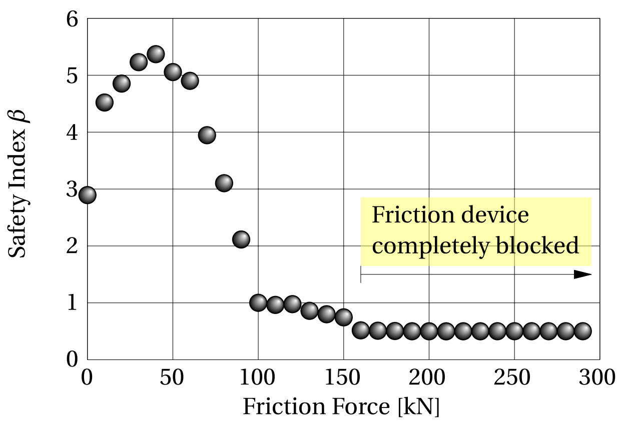

Figure 2: **First passage safety index vs. friction force** s

References

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