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Sensitivity analysis of interfaces using XFEM

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ABSTRACT

The goal of the present contribution is to point out the essential theoretical and computational aspects of structural analysis, sensitivity analysis and structural optimisation for weak discontinuities occurring at interfaces in multi-material structures. In the framework of structural optimisation we propose to minimize stress concentrations at the material interface which may lead to damage and failure in the structure. Consequently, stress based constraint functions must be implemented in the optimisation problem and special attention must be devoted to the reliable stress computation at the boundary. Additionally, geometric modelling of the internal boundary is an important task. The classical parametric approach is based on a CAGD description for the shape of the interface with a conforming finite element mesh. The advantages and disadvantages are similar to the treatment of topology optimisation problems. Therefore, alternative approaches use an implicit formulation for the interface representation on an uniform mesh. The application of the implicit description is already established in the framework of structural optimisation and is already reported in Belytschko [1], Allaire [2], Pedersen [3]. In order to take advantage of the classical and the implicit shape representation we prefer a superelliptic representation of the internal material boundary in combination with the classical CAGD description for the shape of the fixed domain. In the paper the key aspects of the resultant geometric modelling are presented and the consequences for the sensitivity analysis are discussed.

For the computation of the reliable stress at the boundary the physical behaviour must be known. In case of multi-material structures different material properties lead to a continuous stress distribution but to a discontinuous strain distribution along the interface. The paper describes and compares different finite element strategies for the approximation of the displacement, stress and strain fields, which benefit from a non-conforming meshing strategy. Both, theoretical results and selected numerical examples are discussed. Two main strategies are established in the framework of non-conforming meshing methods. The aim of the first strategy is to discretise the subdomains separately and to enforce the continuity condition by either a Lagrange multiplier method, the Nitsche approach or the mortar method. The second strategy admits discontinuous physical behaviour modelled by special finite elements. Hence, a single mesh suffices for modelling the interface problem. In detail, the extended finite element method (XFEM) and the method advocated by Hansbo [4] are examined. The capability of the XFEM to compute the stress accurately for multi-material structures is illustrated on numerical applications. In the

XFEM approach the displacement approximation is enriched by a discontinuous part, which represents the physical discontinuous behaviour of the material. In reference to the kind of the enrichment we distinguish between the conforming and non-conforming XFEM-type. In the conforming XFEM the continuity condition is exactly fulfilled and the enrichment function describes the weak discontinuity exactly. In the non-conforming XFEM-type the continuity condition is introduced in the weak formulation, e.g. by Lagrange-multipliers. In the contribution we represent a novel type of non-conforming XFEM. Herein, the Nitsche approach is used to introduce the continuity condition in the weak formulation. In particular, the details of the discretisation and the performance compared to established XFEM-types are discussed.

The combination of XFEM and the implicit shape representation of matrix-void interfaces by the level set method is previously revisited in Miegroet [5]. Here, XFEM and the superelliptic formulation for multi-material structures (matrix-inclusion structures) are illustrated. The advantages of a combined approach compared to the classical approach in the framework of the structural optimisation are specified. Different computational techniques for the sensitivity analysis of the stresses with respect to small perturbations of the interfaces are mentioned. These results are compared due to the theoretical and computational effort for sensitivity analysis. In detail, the discrete sensitivity analysis using the implicit shape representation is embedded in the classical methods of sensitivity analysis. The advantages and disadvantages of the introduced discrete sensitivity analysis are highlighted and compared to the well established shape sensitivity analysis for CAGD based interface problems.

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