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Multi-point source localization methodology for the advection-diffusion equation

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ABSTRACT

A new methodology for the source term identification in the 4D Eulearian advection-diffusion model is presented. The feature is that the source term is structured as a set of point sources with both unknown locations and intensities. This information has to be reconstructed based on space-time point observations of the model state function. The statement is motivated by the necessity of revealing the atmosphere pollutant sources based on the air quality measurements made by a monitoring system.

The problem is decomposed into two subtasks. The first one is to link the unknown source function Q with M measurement data: i.e. the results of measurements I_m and locations of the measurements p^m in the space-time domain D. It is done with the help of adjoint equations [1]. This step results in projections of the unknown source function Q onto the family of specially constructed adjoint weight functions Ψ_m:

$$\langle \Psi_m, Q \rangle = I_m, m = 1...M,$$

where $\langle ., . \rangle$ is a Cartesian product.

The next task is to reconstruct the source function from its projections. Unlike the other works in this area, we do not seek the unknown source function as a distributed function like the specially constructed localization function from [2] or as the linear combination of weight functions Ψ_m (see e.g.[3]). In our work the following structure of the source function Q is considered:

$$Q(x) = \sum_{s=1}^{S} q_s \delta(x - p^s), \quad x \in D,$$

where $\delta(x - p^s)$ is the source with unit intensity situated in the point p^s of the domain D, $q_s > 0$ is the intensity of the s'th source.

Our approach is based on the interplay of functions in a "physical" domain (sets of point sources) and their representations in the space of all measurement results ("analysis" domain R_+^M). The set of all locations (i.e. the set of all finite subsets of the domain D) is the lattice $L(D)$. As far as the source intensity is unknown, we have to work with locations regardless to the intensities. To accomplish this,

we used the operator that maps a single source with unitary intensity onto a vector in "analysis" domain $A(p^s) \in R_+^M$. The m-th coordinate of $A(p^s)$ represents the result of measurement in the point p^m of the modeled concentration with $Q(x) = \delta(x - p^s)$ i.e. $[A(p^s)]_m := \langle \Psi_m, \delta(x - p^s) \rangle, \quad m = 1...M$. This operator allowed us to define cone-shaped location objects in the "analysis" domain:

$$Cone\{p^1, ..., p^S\} := \left\{ \sum_{s=1}^{S} q_s A(p^s) | q_1 > 0, ..., q_S > 0 \right\} \subset R_+^M, \quad \{p^1, ..., p^S\} \in L(D).$$

Such cone is the same for all the sets of sources located in the set of points $\{p^1, ..., p^S\} \subset D$ with all possible combinations of intensities $q_1, ..., q_S$. In this framework, our ultimate goal is to find all the location cones containing the specific point $I = [I_1, ..., I_M] \in R_+^M$.

To find a sample location we use the iterative computational algorithm that minimizes the functional

$$F(\{p^1, ..., p^S\}) := \min_{q_1, ..., q_s \geq 0} \left\| \sum_{s=1}^{S} q_s A(p^s) - I \right\|^2, \quad \{p^1, ..., p^S\} \in L(D).$$

on the lattice $L(D)$. The procedure of evaluation of $F(\cdot)$ is equivalent to the solution of problem of finding the source intensities provided locations and measurement data. It is essentially a quadratic programming problem.

The solution is built iteratively point by point with two operations: expansion and contraction. The role of the expansion is to construct a cone approximating I with the specified accuracy. The given location $P^N \in L(D)$ is expanded according to the rule:

$$P^{N+1} := P^N + \left\{ \arg\min_{p \in D} \left[F(P^N + \{p\}) \right] \right\}.$$

The minimization can be done either globally (i.e. with evaluation of the function $F(P^N + \{p\})$ for every $p \in D$) or locally (i.e. the minimum is sought iteratively with simultaneous evaluation of the function in some vicinity of a point.

The role of the contraction is to remove insignificant points with respect to the functional $F(\cdot)$. The point $p \in P^{N+1}$ with minimal significance is removed from the location P^{N+1}:

$$P^N := P^{N+1} - \left\{ \arg\min_{p \in P^{N+1}} \left[F(P^{N+1}) - F(P^{N+1} - \{p\}) \right] \right\}.$$

In the current version of the algorithm a number of expansion steps are followed by a number of contraction steps. The combinatorial nature of the algorithm suggests an extensive parallelization which has been implemented in the realization of the both steps of the algorithm.

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