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Grid Computing for the Bi-CGSTAB method Applied to Solution of the Finite-Element Hermite Collocation for Elliptic PDEs

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ABSTRACT

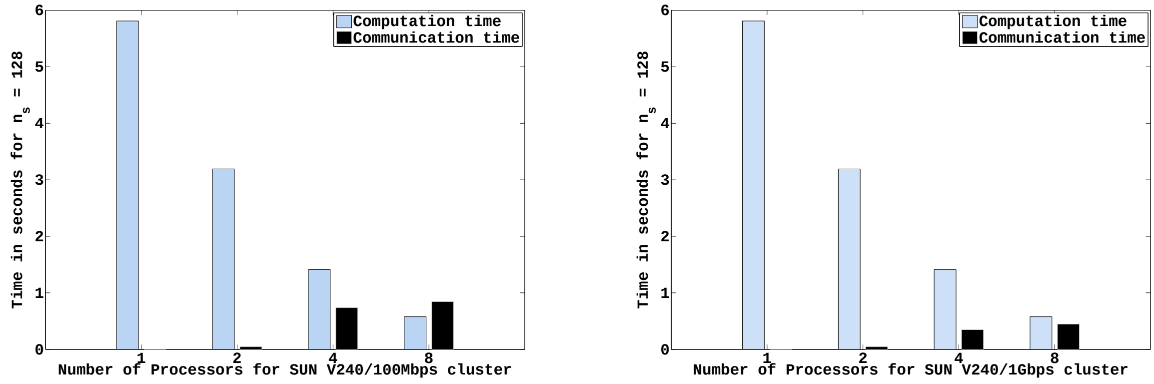
Hermite Collocation is a high order finite element scheme used as a PDE discretizer especially when continuous first derivatives are required. Among other properties, Collocation produces large and sparse systems of equations which poses no pleasant properties (e.g. symmetry). Memory requirements and performance are two of the main factors suggesting the usage of iterative methods on multiprocessor environments [1,2]. This motivated relevant research in the areas of iterative method analysis and parallel algorithm development. Main issues addressed were concerning both algorithmic (multi-color orderings, domain decomposition/partitioning techniques, parallel preconditioners, etc) and architectural (memory management/distribution, processor architecture, etc) aspects.

Working towards this direction, in [3], we have considered the implementation of the SOR and the Bi-CGSTAB [4] iterative methods for solving the Collocation system for the Poisson problem on Distributed-Shared memory (DSM) machines, improving the overall performance by managing the whole computation in order to maximize locality and minimize communication among the processing elements.

The work herein extends the results in [3] by addressing the problem of efficiently managing the intense *Grid computation* involved when the preconditioned Bi-CGSTAB Krylov method is employed for the iterative solution of the large and sparse linear system arising from the discretization of the *Modified Helmholtz-Dirichlet problem*

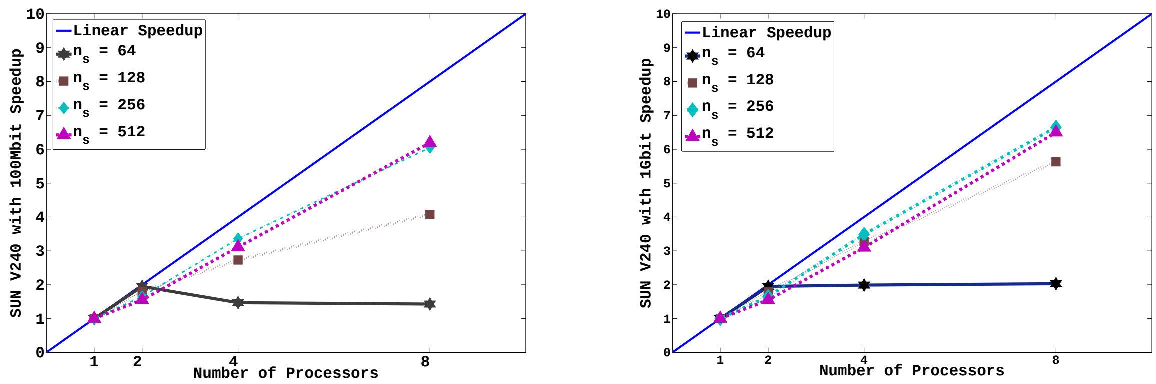
$$\left\{ \begin{array}{rcl} \nabla^2 u(x,y) - \lambda u(x,y) & = & f(x,y)\ ,\ (x,y)\in\Omega \\ u(x,y) & = & g(x,y)\ ,\ (x,y)\in\partial\Omega \end{array} \right. \quad (1)$$

by the Hermite Collocation method. Taking advantage of the Collocation matrix's red-black ordered structure we organize efficiently the whole computation and map it on a pipeline architecture with master-slave communication. Its implementation, through MPI programming tools [5], is realized on a SUN V240 cluster, interconnected through a 100Mbps and 1Gbps ethernet network. To demonstrate its performance we have included results from the numerical experiments conducted, for several values of the parameter λ and discretization sizes n_s, accompanied by computation/communication and speedup measurements (see for example the Figures 1 and 2 that follow).



(a) (b)

Fig. 1 : Time measurements for the SUN V240 cluster using (a) 100Mbps and (b) 1Gbps interconnection.



(a) (b)

Fig. 2 : Speedup measurements for SUN V240 cluster using (a) 100Mbps and (b) 1Gbps interconnection.

REFERENCES

[1] J. Dongarra, I. Foster, G. Fox, W. Gropp, K. Kennedy, L. Toczon and A. White. *SourceBook of Parallel Computing*, Morgan Kaufmann Publishers , 2003.

[2] E. Mathioudakis, E. Papadopoulou and Y. Saridakis. “Iterative solution of elliptic Collocation systems on a Cognitive Parallel Computer”. *Comput. & Maths with Appl.*, Vol. **48**, 951–970, 2004.

[3] E. N. Mathioudakis and E. P. Papadopoulou. “MPI Managment of Hermite Collocation computation on a Distributed-Shared memory system”. *WSEAS Trans. on Mathematics*, Vol. **5**,(5), 520–525, 2006.

[4] H. A. van der Vorst. “Bi-CGSTAB : A fast and smoothly converging variant of Bi-CG for the solution of nonsymmetric linear systems”. *SIAM J. Sci. Statist. Comput.*, Vol. **13**, 631–644, 1992.

[5] Message Passing Interface (MPI) web page, *http://www.mpi-forum.org*.