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# QUASI OPTIMAL PETROV-GALERKIN METHODS FOR HELMHOLTZ PROBLEM

**Abimael F. D. Loula[1] and Daniel T. Fernandes[2]**

Laboratório Nacional de Computação Científica
Av. Getúlio Vargas 333, Petrópolis, RJ - Brasil
[1]aloc@lncc.br, [2]danieltf@lncc.br

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## ABSTRACT

Numerical approximation of time-harmonic acoustic, elastic and electromagnetic wave problems governed by the Helmholtz equation is particularly challenging as reported in a vast literature [1-12]. The oscillatory behavior of the exact solution and the quality of the numerical approximation depend on the wave number $k$. To approximate Helmholtz equation with acceptable accuracy the resolution of the mesh should be adjusted to the wave number according to a rule of thumb [1], which prescribes a minimum number of elements per wavelength. Despite of this rule, the performance of the Galerkin finite element method deteriorates as $k$ increases. This misbehavior, known as pollution of the finite element solution, can only be avoided after a drastic refinement of the mesh, which normally entails significant barriers for the numerical analysis of Helmholtz equation at mid and high frequencies.

There exist several attempts to minimize the phase error of finite element approximations to Helmholtz equation. In one-dimension a Galerkin Least Square (GLS) stabilization, as proposed in [2], can completely eliminate the phase error, but not in two or three dimensions [5] and [8]. For two dimensions, stencils with minimal pollution error are constructed in [4] through the Quasi Stabilized Finite Element Method (QS). Finite element methods based on variational formulations, such as Residual-Based Finite Element Method (RBFEM) [8] and Discontinuous Finite Element Method at Element Level (DGB) [10] and [11], have also been developed to minimize the phase error in two dimensions.

The DGB method is a discontinuous Galerkin finite element formulation with discontinuities introduced locally, inside each element. The discontinuous interpolation functions can be viewed as discontinuous bubbles and the corresponding degrees of freedom can be eliminated at element level by static condensation yielding a global matrix topologically equivalent to those of classical finite element approximations. The free parameters, related to the weak enforcement of continuity inside each element are determined explicitly minimizing the pollution effect. For uniform meshes the DGB stencil with minimal pollution error is identical to QSFEM stencil derived in [4].

In the present work we propose a finite element method for Helmholtz problem in two dimensions based on a Petrov Galerkin formulation. Nearly optimal polynomial weighting functions with local support

are derived aiming at obtaining finite element solutions close to the best approximation [7,13]. To this end we consider bubble functions defined not only on the element level but also on macroelement level. At each node of the mesh, a global basis of the weighting space is obtained adding to the Lagrangian interpolation function linear combinations of the bubble functions defined on the macroelement adjacent to this node. A nearly optimal global weighting function, with the same support of the corresponding test function, is obtained after computing the coefficients of these linear combinations attending some optimality criteria. This is done numerically through a preprocessing that can be applied to any finite element mesh. For uniform mesh a quasi optimal interior stencil is obtained with this preprocessing. The method is naturally applied to non uniform or unstructured meshes.

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