8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineeering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

Multigrid Methods for Inhomogeneous Problems in Solid Mechanics

*** Johann Bitzenbauer**[1], **Karl Schweizerhof**[2]

[1] Institute for Mechanics
University of Karlsruhe (TH)
Kaiserstr. 12
D-76131 Karlsruhe
Germany
bitzenbauer@ifm.uni-karlsruhe.de
http://www.ifm.uni-karlsruhe.de

[2] Institute for Mechanics
University of Karlsruhe (TH)
Kaiserstr. 12
D-76131 Karlsruhe
Germany
schweizerhof@ifm.uni-karlsruhe.de
http://www.ifm.uni-karlsruhe.de

Key Words: *Solid Mechanics, Nonlinear Elasticity, Finite Elements, Multigrid Methods.*

ABSTRACT

Multigrid methods are efficient and fast solution schemes for discretized partial differential equations, their construction is based on the discretization of the problem on different grids. In many practical applications, their efficiency is suffering because suitable coarse scale discretizations are no longer available, i.e. as in the case of complicated curved boundaries or discontinuous coefficients. On the other hand, every classical finite-element-scheme resolving small geometric details in a correct manner, necessarily leads to large systems of linear equations. Nonmultilevelbased solution strategies are in general not recommended then.

In [1-3] finite element spaces are introduced, for which the minimal dimension of the discretization is no longer coupled with a possibly complex geometry. These so-called Composite Finite Elements (CFE) are defined using a hierarchy of grids. In the multigrid context, they are very well suited for the construction of a sequence of appropriate coarse level discretizations. First usage of the CFE-method was for problems with complicated curved boundaries, a generalization to problems with discontinuous coefficients is presented in [4]. In contrast to purely algebraic multigrid methods, here geometrical information is used for the construction of the intergrid transfer operators. For this, a hierarchically nested sequence of grids is required, which should be constructed during mesh generation.

On the other hand, in a lot of situations meshing possibly already was done and a given fine scale discretization has to be used. A priori no grid hierarchy for coarse scale discretizations is available then. If the link between the geometry and the algebraic system of linear equations arising from the finite elemente discretization is known, quadtree/octree-type algorithms [5] can be used to construct suitable auxiliary grids for interpolation. This algorithm can be regarded as a mixed geometric/algebraic multigrid-method [6].

In the contribution, robust multigrid solvers for the discretization of the equations of nonlinear solid mechanics on complicated domains will be presented. The focus is on the transfer of existing algorithms for 2D-Poisson-type problems to two- and threedimensional nonlinear elasticity [7]. Numerical experiments demonstrate the efficency and the reliability of the extended CFE-concept.

REFERENCES

[1] W. Hackbusch and S.A. Sauter. “A new Finite Element Approach for Problems Containing Small Geometric Details”. *Arch. Math.* Vol. **34**, 105–117, 1998.

[2] W. Hackbusch and S.A. Sauter. “Adaptive Composite Finite Elements for the Solution of PDEs containing non-uniformely distributed Micro-Scales”. *Mat. model.* Vol. **8**, 31–43, 1996.

[3] W. Hackbusch and S.A. Sauter. “Composite finite elements for problems containing small geometric details, Part II: Implementation and numerical results”. *Comp. Vis. Sci.*, Vol. **1**, 15–25, 1997.

[4] S.A. Sauter and R. Warnke. “Composite Finite Elements for Elliptic Boundary Value Problems with Discontinuous Coefficients”. *Computing*, Vol. **77**, 29–55, 2006.

[5] H. Samet. *The Design and Analysis of Spatial Data Structures*, Addison-Wesley, 1990.

[6] D. Feuchter, I. Heppner, S.A. Sauter and G. Wittum. “Bridging the gap between geometric and algebraic multi-grid methods”. *Comp. Vis. Sci.*, Vol. **6**, 1–13, 2003.

[7] J. Bitzenbauer and K. Schweizerhof. “Mehrskalenberechnungen bei inhomogenen Körpern”. *PAMM* Vol. **6**, 211–212, 2006.