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A multiscale approach for the modelling of propagating discontinuities

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ABSTRACT

Motivation

The multiscale modelling of heterogeneous materials is often based on unit cells, which allow for the computation of effective material properties by means of a homogenization process. The failure modes of such materials are governed by several mesomechanical effects, like debonding and crack propagation. Therefore, the macroscopic description of the material behaviour should also include the mesoscopic failure processes. Recently, different methods for the multiscale modelling of failure have been presented. In [1] and [2] FE^2 approaches for the modelling of localized failure were introduced, which circumvent the loss of material stability by introducing discontinuities on the macroscale. In [3] a multiscale extended finite element method based on a domain decomposition method is described, which introduces discontinuities on the micro- and the macroscale.

This contribution presents a multiscale approach for the propagation of discontinuities, which invokes unit cell computations in the vicinity of the crack tip. Thus, the influence of the mesostructure on the propagation of the crack is taken into account. Once the unit cell loses all strength due to the propagating discontinuities, an equivalent discontinuity on the macroscale is injected. Thus, the explicit simulation of the mesoscale is limited to the vicinity of the crack tip.

Multiscale Approach

In the present contribution a two-scale approach is considered. The computation of crack propagation on the macroscale is enhanced by unit cell computations on the mesoscale. This multiscale approach can be described as follows: the macroscopic structure is divided into subdomains (the macroscopic finite elements), which can be linked to unit cells. This is usually done in critical subdomains, where high strain gradients appear, e.g. in the vicinity of the macroscopic crack tip. The macroscopic computation yields a deformation gradient $\boldsymbol{F}^M$ at each Gauss point, which is transferred to the unit cells in the form of displacement boundary conditions:

$$\boldsymbol{x}^m(\partial\Omega^m) \,=\, \boldsymbol{F}^M \,\cdot\, \boldsymbol{X}^m(\partial\Omega^m),$$

whereby ^{m} denotes mesoscopic and ^{M} macroscopic quantities, $\boldsymbol{X}^m$ are the coordinates in the material configuration and $\boldsymbol{x}^m$ in the spatial configuration.

The boundary value problem of the unit cell will then be solved using certain constitutive laws and taking into account the propagation of discontinuities and provides both the stress response at the

macroscale and in some cases the orientation and jump of the macroscopic discontinuity. The relation between the macroscopic deformation gradient $\boldsymbol{F}^M$ and the volume average deformation gradient $< \boldsymbol{F}^m >$ of the unit cell is given by

$$< \boldsymbol{F}^m >= \boldsymbol{F}^M + \frac{1}{|\Omega^m|} \int\limits_{\Gamma} [\![\boldsymbol{\varphi}]\!] \otimes \boldsymbol{N} \, dA,$$

whereby $[\![\boldsymbol{\varphi}]\!]$ denotes the jump and $\boldsymbol{N}$ the normal of the discontinuity. The macroscopic Piola stress is equal to the volume average of the Piola stresses on the mesoscale. When the unit cell loses its strength due to the propagating discontinuities, an equivalent discontinuity is introduced on the macroscale and the explicit simulation of the unit cell is not necessary anymore. The direction and the magnitude of the macroscopic discontinuity are provided by the unit cell computation. The general outline of the multiscale approach is outlined in figure 1.

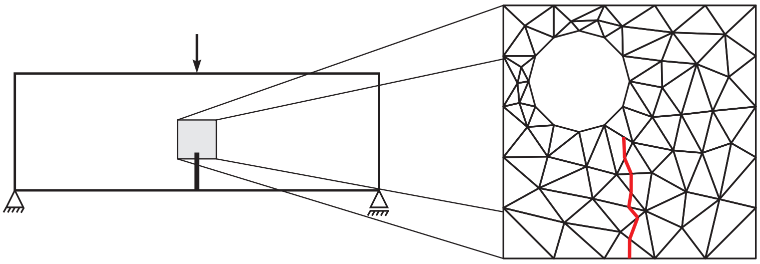

Fig. 1: General setup of the considered method

Simulation of Discontinuities

In the present approach discontinuities can occur on both meso- and macroscale. In the framework of the finite element method the simulation of propagating discontinuities creates meshing difficulties. Here, the propagation of discontinuities is described independently of the finite elements by the introduction of discontinuous elements, following an approach introduced in [4]. On the mesoscale a crack propagation criterion based on maximum stresses is applied to decide whether the crack propagates and in which direction. The discontinuity on the macroscale is introduced when the unit cell fails. Its direction is then prescribed by the solution of the unit cell calculation. The discontinuities on the macroscale are introduced elementwise such that a continuous crack path is obtained. The applicability of the present method to model and simulate crack propagation in heterogeneous materials will be highlighted by numerical examples.

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