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# Inverse Finite Element Method for Real-Time Structural Health Monitoring: Application to Fiber-Optic Strain Measurements

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## ABSTRACT

A study focused on developing suitable computational schemes for designing optimally distributed fiber-optic strain sensors is discussed in the context of the Inverse Finite Element Method for reconstruction of full-field displacements in plate structures undergoing in-plane, bending, and twisting modes of deformation.

Monitoring the structural health of aerospace structures is an enabling technology for the next generation of aerospace vehicles and an important prerequisite for improved aviation safety. Effective structural health monitoring requires an adequate distribution of sensors on the structure that would enable accurate real-time assessment of structural integrity. For real-time structural strain measurements during operational conditions, Fiber-Optic Strain Sensing (FOSS) technology offers numerous benefits, including low-weight and immunity to electromagnetic interference and moisture. Moreover, application of the Optical Frequency-Domain Reflectometry (OFDR) technique makes it possible to have thousands of fiber-optic strain sensors embedded on a single optical fiber.

A computational mechanics methodology which is capable of reconstructing the full-field structural displacements, strains, and stresses based upon discretely distributed strain sensor measurements, known as the Inverse Finite Element Method (iFEM), has been recently developed at NASA Langley Research Center [1-3]. The iFEM has been demonstrated to be a high-fidelity computational tool for use in real-time applications, such as providing feedback to the actuation and control systems of adaptive self monitoring structures. The results in [1-3] showed excellent agreement between reference (or measured) and iFEM reconstructed displacement fields, with optimal solutions obtained when strain rosettes were placed at the centroids of the inverse finite elements.

This paper presents a generalized formulation of a least-squares inverse finite element method developed by Tessler and Spangler [1-3]. The error functional uses the least-squares-difference terms comprised of the Mindlin-theory strain measures that are expressed in terms of the displacements and the corresponding strain measures computed from the experimental strains. Within the present formulation, all strain compatibility relations are explicitly satisfied, and neither elastic nor inertial material constants are used to reconstruct the displacement field. A three-node, inverse-shell element is developed having six conventional degrees of freedom at each node, i.e., three displacements and three rotations. The kinematic variables are interpolated

using the lowest-order anisoparametric (interdependent) $C^0$-continuous shape functions, i.e., linear in-plane displacements and bending rotations, and a constrained type quadratic transverse displacement.

The present study addresses two key issues associated with the utilization of measured strain data in the context of the iFEM and FOSS technologies:

1) The most common application of the FOSS technology provides strain measurements corresponding to a single strain component in the direction of the fiber. As a consequence, suitability of the iFEM methodology for the high-fidelity reconstruction of a complete three-dimensional displacement field in the presence of an "incomplete strain measure" (where only one strain component is known at a given strain-sensor location) needs to be examined.
2) The influence of strain-sensor placement on the accuracy of the iFEM methodology. This is the first step toward the optimization of strain-sensor placements.

In the present formulation, each of the least-squares norms corresponding to the eight strain measures of Mindlin theory employs an independent weighting parameter. This enables physically meaningful least-squares matching of the interpolated and measured strains in accordance with the magnitude of the measured strain data. Thus, when only an incomplete measured strain set is available, the missing measured strains are set to zero. Accordingly, the corresponding weighting parameters are assigned very small values, thus discarding the influence of the missing strain data on the reconstructed deformation pattern. This approach allows iFEM to be used effectively even when only few elements in the discretization have measured strain data, i.e. when the number of sensors is less than the number of inverse shell elements. Moreover, adequate solutions for problems for which the strain sensors provide only a single strain component at a given location will be obtained. To allow for a convenient computational environment, data handling, and processing of results, the present iFEM formulation has been implemented using MATLAB®.

Extensive numerical studies will be presented to evaluate the effectiveness and computational efficiency of the generalized iFEM methodology for plate bending problems in the context of incomplete strain measurements. Moreover, application of iFEM coupled with suitable optimization algorithms will be undertaken to address the issues of optimal strain-sensor locations.

## REFERENCES

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