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A Space-Time hp-Adaptive Scheme for Evolution Equations

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ABSTRACT

In this talk we present an hp-adaptive scheme in space and time for the discretization of systems of evolution equations based on the higher order Discontinuous Galerkin method in space and multistep methods in time. We use h-adaptivity, i.e., general grid structures with non-conform adaptivity and local time stepping, to achieve a high degree of efficiency. Since our focus is on systems of balance laws, we discuss approaches for gradient limiting and p-adaptivity for stabilizing the scheme in the regions of discontinuities. The basis of our scheme is an a-posteriori error estimate for the semi-discrete method which will be briefly discussed and compared with a heuristic approach. For the implementation of the scheme we use the software environment DUNE.

Many applications require the solution of complex non-linear systems of evolution equations in divergence form

$$\partial_t U(t,x) + \nabla \cdot \big(F(U(t,x),t,x) + a(U(t,x),t,x)\nabla U(t,x) \big) = S(U(t,x),t,x)\ . \qquad (1)$$

Depending on the application governed by this system, both a high degree of accuracy and a high degree of efficiency have to be a central aspect in the design of the numerical scheme. These goals can be achieved on the one hand by a careful reduction of the complexity of the mathematical model and on the other hand by using local grid adaptivity and parallelization strategies. If possible these techniques should be simple to use for the developer and a modification of the mathematical model should not require a major redesign of the code. In the software library DUNE-FEM [1] both the required flexibility and efficiency is achieved by using generic software design techniques based on static interfaces implemented in C++. The basis of this project is the Discontinuous Galerkin method using implicit/explicit Runge-Kutta methods for the time discretization. DUNE-FEM is based on the grid interface concept developed in the DUNE project [2]. Here general concepts for using locally adapted grids in a parallel environment are developed.

The Discontinuous Galerkin method is a higher order discretization method for evolution equations of the form (1), which can be easily adapted to both the diffusion and the advection dominated case [3]. Due to its small stencil, the method can be easily implemented on non-conform grid structures and thus simplifies the use of local adaptivity and parallelization based on domain decomposition. In the case of small or zero viscosity the higher order versions of the Discontinuous Galerkin method become unstable and some limiting of gradients is required. Here unstructured grids prove to be difficult; while on structured Cartesian grids minmod type limiting is very effective [3], limiters on unstructured grids either lead to a severe degeneration of the approximation in smooth regions or are unstable in extreme cases.

In [4] we present an a-posteriori error estimate for the higher order Discontinuous Galerkin method for scalar balance laws

$$\partial_t u(t,x) + \nabla \cdot f(u(t,x)) = 0 \,. \tag{2}$$

The result holds for the semi-discrete scheme on arbitrary grid structures and is heuristicly extended to a fully discrete scheme by using explicit Runge-Kutta methods for the time discretization. The error estimate is then used to locally reduce the order of the scheme (p-adaptivity) and simultaneously the local grid resolution is adapted (h-adaptivity) - thus the stability and efficiency of the scheme is achieved by hp-adaptivity in space.

In this talk we present the DUNE project and the software techniques used for solving evolution equations of the form (1). We then briefly recall the results from [4] and present a method for extending the hp-adaptive scheme also to the time discretization by using local time stepping based on a class of multistep methods.

REFERENCES

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