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On the choice of abstract projection vectors for second level preconditioners

*** C. Vuik[1], J.M. Tang[2] and R. Nabben[3]**

[1] Delft University of Technology
Mekelweg 4
2628 CD Delft
Netherlands
c.vuik@tudelft.nl
http://ta.twi.tudelft.nl/nw/users/vuik/

[2] Delft University of Technology
Mekelweg 4
2628 CD Delft
Netherlands

[3] Technische Universität Berlin
Strasse des 17. Juni 136
Berlin
Germany

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ABSTRACT

In the literature, there are various strategies known to choose projection vectors, which can be used in projection methods known in the field of deflation, DDM or MG methods. Each field has its typical strategy. We describe them concisely below.

In the field of deflation (and also (A)MG), we typically use approximations of eigenvectors associated with unfavorable eigenvalues (often the small eigenvalues), see for example Chapman & Saad [2], Burrage *et al.* [1] and Saad *et al.* [6]. They are often dense, so that it is only efficient if the number of these vectors is small.

In the field of DDM, projection vectors based on subdomains are used, see for example Vuik *et al.* [8] and Nicolaides [4]. The number of projection vectors can now be relatively large since the vectors are sparse. The relation with projection vectors based on eigenvectors is very strong. It can be shown that projection vectors based on subdomains approximate slow-varying eigenvectors. These slow-varying eigenvectors often correspond to small eigenvalues which cause the slow convergence of the iterative process.

In MG, the projection vectors are typically (sparse) interpolation/restriction operators for transfer between coarse and fine grids. Typical examples of MG projection vectors can be found in for example the books of Widlund & Toselli [7] and Wesseling [9]. We can think of vectors which are determined geometrically or algebraically. Also in this case, these vectors are related to those known in the other fields, since they have often the aim to get rid of slow-modes.

In addition to the projection vectors known in deflation, MG and DDM, there are more strategies known. For example, projection vectors based on (previous) solutions are used by Clemens *et al.* [3] or recycling information based on (previous) Krylov iterations, see the works of De Sturler et al. [5]. Generally, we can say that an additional projection technique, using a user's favorite strategy of choosing projection

vectors, should always be included in a Krylov solver to get rid of all unfavorable eigenvalues that cause deterioration of the convergence of the iterative process.

In this paper we give a comparison of projection vectors based on subdomains and physically motivated (level set) projection vectors. In this comparison we vary the number of projection vectors and measure the CPU time and the number of iterations. Furthermore, we investigate how the convergence of the method depends on the ratio of the discontinuous coefficients.

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