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# ON THE ANALOGY BETWEEN STRUCTURAL LAYOUT OPTIMIZATION AND PLASTIC LIMIT ANALYSIS FORMULATIONS

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**ABSTRACT**

The analogy between certain plastic limit analysis and structural optimization problems has been known for at least half a century[1,2]. For example, the arrangement of slip-line discontinuities in plane strain metal plasticity problems is known to mirror the arrangement of bars in optimal ‘Michell’ trusses (the geometry of both are Hencky-Prandtl nets, which are orthogonal curvilinear co-ordinate systems).

However, increasing specialization amongst researchers means that there is perhaps less cross-fertilization between subject areas nowadays, and consequently the significance of the analogy appears to have been overlooked by many. This has led to analytical solutions for problems already identified as being analagous being derived separately (e.g. the plane strain compressed block limit analysis problem involving a Tresca material and rough platens is analagous to a well known Michell cantilever problem, yet solutions were derived separately in each case by Chakrabarty[3] and Lewinski et al.[4]).

Furthermore, given the current importance of computer based procedures it is perhaps even more significant that the numerical methods developed in each of the parallel fields are frequently not transferred to the other. This was recently partially addressed by the authors who realised that a Linear Programming (LP) formulation developed some decades ago to identify the optimal layout of bars in trusses[5] could be applied to plastic limit analysis problems. The resulting procedure, Discontinuity Layout Optimization (DLO), involves identifying discontinuities which form in a body at failure and shows considerable promise, being more generally applicable than some previously proposed numerical procedures (e.g. Dewhurst and Collins[6]). Table 1 shows the main features of the analogy (after Smith and Gilbert[7]).

Although the basic analogy only applies to plane strain limit analysis problems involving translational failure mechanisms and a Tresca material, with relatively minor modification it is for example possible to analyse problems involving the sort of frictional materials frequently encountered by civil and structural engineers (assuming a Mohr-Coulomb material). DLO can therefore be used to identify failure mechanisms involving foundations, slopes and/or retaining walls, usefully bridging the gap in complexity between commonly applied hand-type procedures (relying on closed form analytical solutions) and more complex incremental elasto-plastic procedures. DLO can also be applied to reinforced concrete slab problems, providing a useful means of systematically identifying critical yield line patterns. A selection of DLO results for standard problems are tabulated in Table 2. In all cases nodes were

| | Truss problem | Slip-line discontinuity problem |
|---|---|---|
| LP problem variables | Internal bar forces: $\mathbf{q}$ | Slip displacements: $\mathbf{d}$ |
| Governing coefficient matrix | Equilibrium: $\mathbf{B}$ | Compatibility: $\mathbf{B}$ |
| Applied loads / displacements | External loads: $\mathbf{f}$ | Nodal displacements: $\mathbf{u}$ |
| Objective function | Minimise volume $V$ | Minimise work $E$ |
| Graphical analysis method | Maxwell force diagram | Hodograph (velocity diagram) |

Table 1: Features of analogy between plane truss layout optimization (equilibrium formulation) and slip-line discontinuity layout optimization (displacement formulation)

| Problem | Literature solution | DLO solution | % diff. |
|---|---|---|---|
| Square concrete slab (simple supports) | 24 | 24 | 0.00 |
| Square concrete slab (fixed supports) | 42.851 | 42.869 | 0.04 |
| Compressed metal block (width / height = 3.64) | 3.334 | 3.335 | 0.03 |
| Smooth foundation footing on sand ($N_q$ with $\phi = 25^o$) | 10.662 | 10.684 | 0.21 |
| Smooth foundation footing on sand ($N_\gamma$ with $\phi = 25^o$) | 3.461 | 3.563 | 2.94 |

Table 2: Comparison of literature and DLO solutions for various well known problems (solutions are expressed in the form commonly used for each problem type, e.g. as a dimensionless load factor)

laid out uniformly over the problem domains (i.e. no manual, problem specific, refinement was undertaken). The close agreement with known exact analytical solutions using the basic DLO procedure demonstrates its promise (solutions are within 1% in all but one case).

There appears to be scope for considerably more cross-fertilization in the future. For example, finite element limit analysis procedures have developed rapidly in the past few years and, when the applied to analogous structural optimization problems, might prove useful to structural optimization researchers. Hopefully this contribution will stimulate others to explore such opportunities further.

## REFERENCES


[1] W.S. Hemp. *Rep. 115: Theory of structural design*. College Aeronautics, Cranfield, 1958.

[2] W. Prager. "On a problem of optimal design", *Proc. IUTAM Symposium on non-homogeneity in elasticity and plasticity, Warsaw*, London: Pergamon. 125–132, 1959.

[3] J. Chakrabarty. "The analytical solution of some boundary value problems in plane plastic strain". *Int. J. Mech. Sci.*, Vol. **33**, 89–99, 1991.

[4] T. Lewinski, M. Zhou and G.I.N. Rozvany. "Extended exact solutions for least-weight truss layouts - part i: cantilever with a horizontal axis of symmetry". *Int. J. Mech. Sci.*, Vol. **36**, 375–398, 1994.

[5] W.S. Dorn, R.E. Gomory and H.J. Greenberg. "Automatic design of optimal structures", *J. de Mechanique*, Vol. **3**, 25–52, 1964.

[6] P. Dewhurst and I.F. Collins. "A matrix technique for constructing slip-line field solutions to a class of plane-strain plasticity problems", *Int. J. Numer. Meth. Engng*, Vol. **7**, 357–378, 1973.

[7] C.C. Smith and M. Gilbert. "Application of discontinuity layout optimization to plane plasticity problems", *Proc. R. Soc. A*, Vol. **463**, 2461–2484, 2007.