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# A MULTI-SCALE ANALYSIS OF WOOD PHYSICAL PROPERTIES BY THE REINFORCED-MATRIX PRINCIPLE; FORMULATION BY MORI-TANAKA THEORY

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## ABSTRACT

Wood xylem fiber cell consists of two main parts: a thick secondary wall and thin compound middle lamella (CML). A middle layer (S2) is the thickest layer of the secondary wall and is reinforced with fibrous polysaccharide crystals oriented more or less parallel to the fiber cell axis. The orientation angle (microfibril angle; *MFA*) determines the mechanical properties of the wood xylem fibers, including the longitudinal Young's modulus, anisotropic drying shrinkage, surface growth stress, longitudinal tensile creep deformation, and so forth. Consequently, the S2 layer takes an important role in determining the macroscopic properties of clear wood specimens, with the mechanical interaction between the reinforcing polysaccharides and the encrusting matrix substance ultimately controlling the material properties of the wood.

Any formulation of the material properties of a wood based on its structural hierarchy starts by considering the mechanical properties of the two-phase structure of the cell wall layer; that is, the reinforcing fibrous polysaccharide and the amorphous matrix (Fig.1). The reinforced-matrix hypothesis was originally proposed by Barber and Meylan [1] and provides a theoretical description of the mechanical interaction between these two phases. This model can be expressed using tensor equations:

$$\sigma^W = \sigma^f + \sigma^m, \quad \text{and} \quad \varepsilon^W = \varepsilon^f = \varepsilon^m. \qquad (1)$$

A physical interpretation of the tensorial quantities $\sigma^W$, $\sigma^f$, $\sigma^m$, $\varepsilon^W$, $\varepsilon^f$, and$\varepsilon^m$ in Eq. 1 can be formulated as follows. The fibrous components of the polysaccharides are dispersed uniformly in each cell wall layer to form the framework fibril bundle. Similarly, the lignin–hemicellulose compound is diffused in each layer to provide the isotropic matrix skeleton. It is postulated that the framework bundle and the matrix skeleton occupy the same domain at the mesoscopic viewpoint (layer level). As a consequence, $\sigma^W$ can be regarded as the stress tensor in the cell wall fragment as the whole, and $\sigma^f$ and $\sigma^m$ as the stress tensors in the polysaccharide framework bundle and matrix skeleton, respectively, with $\varepsilon^W$, $\varepsilon^f$, and $\varepsilon^m$ being the respective strains.

However, neither Eq. 1 nor these physical interpretations of $\sigma^W$, $\sigma^f$, $\sigma^m$, $\varepsilon^W$, $\varepsilon^f$, and $\varepsilon^m$ have been defined in any rational manner. Our study aims to resolve this issue using the classical theory of micromechanics developed by Eshelby [2,3] and Mori and Tanaka [4]. We then compare our formulation with that of Cave [5] who proposed a

constitutive relationship for the lignocellulosic material of the wood cell wall on the basis of Hill's theory [6], with the ultimate goal of validating the reinforced-matrix hypothesis as one of the basic theories of cell wall mechanics and physics. Moreover, the formulation, combined with a fiber mechanical model, allows to describe the multi-scale behavior of wood, including (1) generation of maturation strain, (2) hygro-expansion of swelling wood, (3) moisture-dependency of wood elasticity, (4) tension wood gelatinous fiber behavior, and so forth.

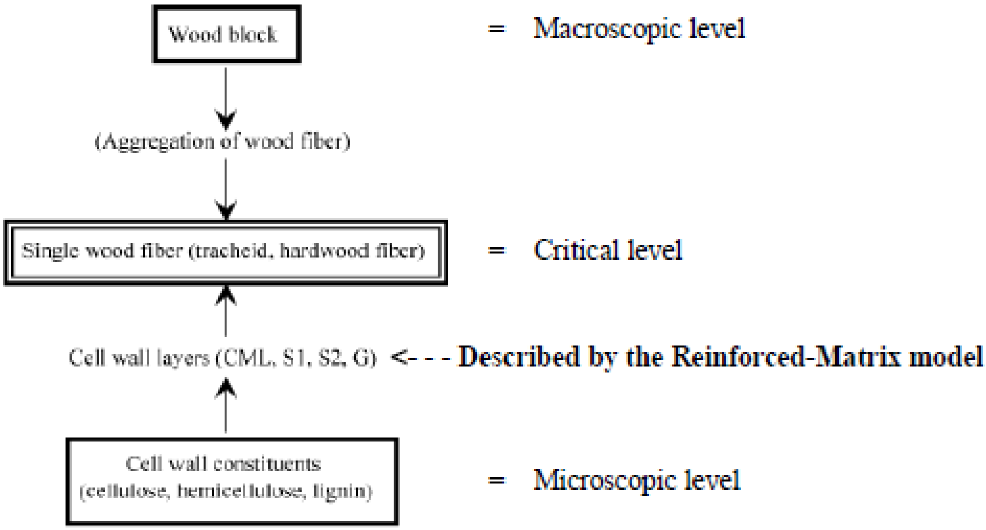


**Fig. 1.** Multi-scale analysis of wood by using a fiber mechanical model as a critical level of the hierarchical structure of wood.

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