8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineeering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

RELIABILITY ANALYSIS OF THE BUCKLING OF IMPERFECT SHELLS BASED ON THE KARHUNEN-LOEVE EXPANSION

*** C. Noirfalise[1], J.-M. Bourinet[1], M. Fogli[1] and B. Cochelin[2]**

[1] Laboratoire de Mécanique et Ingénieries
Aubière, France
Claudine.Noirfalise@ifma.fr,
Jean-Marc.Bourinet@ifma.fr,
Michel.Fogli@ifma.fr

[2] Laboratoire de Mécanique et d'Acoustique
Marseille, France
Bruno.Cochelin@ec-marseille.fr

Key Words: *Thin shells, buckling, random imperfections, Karhunen-Loève expansion, subset simulation, Asymptotic Numerical Method.*

ABSTRACT

Discrepancies observed in experimental buckling loads of shells are mostly due to the random nature of initial shape and thickness imperfections, material parameters and boundary conditions. For this reason, it is of paramount importance to consider this problem from a probabilistic point of view. A few works were carried out on this subject in the past, but they were restricted either to reliability analyses based on closed-form buckling solutions or to finite element (FE) models with modal or quasi-modal imperfections [1]. Other probabilistic approaches based on 3D FE models with Monte-Carlo simulations or stochastic finite elements can be found in [2,3]. The present paper brings a contribution to this specific area and addresses the stability of thin cylindrical shells with random shape imperfections under external pressure. The main purpose of this paper is to show that computing reliability of such uncertain structures based on FE solutions requires efforts at different levels. First of all, a specific attention should be paid for a realistic modeling of random imperfections, based on experimental measurements. Besides, buckling loads are required to be calculated very accurately for reliability studies and it is also expected that the FE code gives robust estimations of these loads, for any realization of the stochastic model. Since a FE solution is involved, the reliability problem is rather computationally demanding and proper algorithms and computational strategies need to be set up in addition.

The present work is based on initial imperfections of 13 cylindrical shells from the Imperfection Data Bank initiated by J. Arbocz [4], which gives radial amplitude of initial imperfections expressed as a Fourier series expansion. Considering the Fourier coefficients as Gaussian random variables, initial shape imperfections are modeled as a two dimensional Gaussian stochastic process and are approximated by a Karhunen-Loève (KL) expansion. This expansion is based on the spectral decomposition of the covariance function of the random process. Two different covariance functions are studied and compared. The first one is directly obtained from the random Fourier series expansion of experimental results. The second covariance function is of exponential type with two correlation lengths, along axial and circumferential directions respectively. In both cases, the KL expansion is determined by solving the Fredholm integral equation with a Galerkin-type numerical procedure.

It is assumed here that small probabilities of failure are to be sought for since a high level of safety is required for engineering structures prone to buckling. In order to solve this rather challenging problem, the present work makes use of the subset simulation method introduced by S.-K. Au and J.L. Beck [5]. This method outperforms crude Monte Carlo in estimating small probabilities of failure, as it converts the initial problem to another more tractable one which basically consists in finding a few larger conditional probabilities of intermediate failure events. It is also useful to point out that approximate methods such as FORM do not converge here, which makes simulation methods the most viable solution for the present problem. The implementation takes advantage of a distributed computing strategy on a multi-processor computer platform. This is a key point here as many calls to a nonlinear FE code are involved, in order to obtain an estimate of acceptable accuracy for the probability of failure.

A major objective of the present study bas been to base it on FE solutions of the buckling problem, so as to provide the reliability algorithm with trustworthy and accurate mechanical results. The present approach makes recourse to a non-conventional FE method, known as the Asymptotic Numerical Method (ANM), which has proved to be both efficient and robust in computing buckling loads [6]. The ANM differs from incremental-iterative methods used in most conventional FE codes. The basic idea is to transform the initial nonlinear problem into a sequence of linear problems by means of asymptotic expansions of displacements, stresses and load factor with respect to a well-chosen parameter. Each linear problem is then solved using a classical FE method.

The results obtained for the cylindrical shell under study for both covariance functions are presented, for a range of service pressures. It is worth mentioning that estimating small probabilities of failure needs very accurate FE predictions of buckling loads as a prerequisite, for a sound basis in structural reliability analysis. This is achieved here thanks to the robust and efficient ANM, combined with an adequate FE mesh which ensures accurate buckling loads for imperfections under consideration. Further works will consist in improving the random model of initial imperfections, by using bounded two dimensional Gaussian stochastic processes such as a Wiener process. Another ongoing axis of research aims at reducing the number of calls to the FE code, for the same level of accuracy obtained in a subset simulation, by applying a Support Vector Machine-based learning to the subset simulation concept [7].

REFERENCES

[1] J.-M. Bourinet, N. Gayton, M. Lemaire and A. Combescure. "Reliability analysis of stability of shells based on combined finite element and response surface methods". *IASS-IACM 2000*, Chania, Crete, Greece, 5-7 June 2000.

[2] C.A. Schenk and G.I. Schuëller. *Uncertainty assessment of large finite element systems*, Springer Verlag, New York, 2005.

[3] V. Papadopoulos and M. Papadrakakis. "The effect of material and thickness variability on the buckling load of shells with random initial imperfections". *Computer Methods in Applied Mechanics and Engineering*, Vol. **194**, 1405–1426, 2005.

[4] J. Arbocz and H. Abramovich. *The initial imperfection data bank at the Delft University of Technology*, Part VI, Memorandum M-867, Delft University of Technology, 1999.

[5] S.-K. Au and J.L. Beck. "Estimation of small failure probabilities in high dimensions by subset simulation". *Probabilistic Engineering Mechanics*, Vol. **16**, 263–277, 2001.

[6] B. Cochelin. "A path-following technique via an asymptotic-numerical method". *Computers and Structures*, Vol. **53**, 1181–1192, 1994.

[7] F. Deheeger, J.-M. Bourinet and M. Lemaire. "Active learning based on subset simulation and Support Vector Machines for small probabilities of failure: ^{2}SMART method". In preparation.